UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Axiomatic Set Theory

(Axiomatische Verzamelingentheorie)

2002/2003; 3rd Trimester
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Homework Set # 1 Deadline: Thursday, April 24th, 2003

Exercise 1.1

Prove that $(\mathsf{Aus}) + (\mathsf{Un}) + (\mathsf{Pair}) + (\mathsf{Ext})$ imply EST.
In other words, as soon as we have (Aus), we can drop $(\varnothing)$ for the list of axioms.
(*Hint.* Note that $\exists x(x = x)$ is a theorem of predicate logic alone.)

Exercise 1.2

Prove in the theory $\mathsf{EST} + (\mathsf{Aus}) + (\mathsf{Pow})$ that the sets

$$\begin{aligned}
\mathrm{Fun}(X,Y) &:= \{f\,;\, f \text{ is a function from } X \text{ to } Y\},\\
\mathrm{Inj}(X,Y) &:= \{f\,;\, f \text{ is an injection from } X \text{ to } Y\},\\
\mathrm{Sur}(X,Y) &:= \{f\,;\, f \text{ is a surjection from } X \text{ to } Y\},\\
\mathrm{Bij}(X,Y) &:= \{f\,;\, f \text{ is a bijection from } X \text{ to } Y\}.
\end{aligned}$$

exist.

Exercise 1.3

Prove (informally) that EST has no finite models.
(*Hint.* Do this by taking an arbitrary model of EST and defining an injection from the natural numbers into that model.)

Exercise 1.4

Prove in $\mathsf{EST} + (\mathsf{Ers})$ that for all sets X and Y, the cartesian product $X \times Y$ exists. (Do not use (Pow).)
(*Hint.* Prove first that for all X and Z, the set $X \times \{Z\}$ exists, then look at the class term F_X mapping Z to $X \times \{Z\}$ and the image of Y under F_X.)

Exercise 1.5

Prove that $\mathsf{EST} + (\mathsf{Ers})$ implies (Aus).