Introduction to symplectic geometry

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Summary of exam topics

1. Linear symplectic geometry
   (a) the different types of linear subspaces and their normal forms
   (b) linear symplectic maps and their properties
   (c) relations between $\mathrm{Sp}(2n)$, $U(n)$ and $O(2n)$
   (d) Maslov indices for loops of symplectic matrices and of Langrangian subspaces: definitions, properties and computations in examples
   (e) compatible complex structures, contractibility of $\mathcal{J}(\omega_{\mathrm{st}})$
   (f) relation to hermitian metrics
2. Basic on symplectic manifolds
   (a) basic definitions and examples
   (b) symplectic and Hamiltonian diffeomorphisms, including basic properties such as Poincare reccurence
   (c) Moser's argument with applications, especially Darboux' theorem
   (d) special submanifolds, examples of normal forms for neighborhoods
   (e) contact manifolds, symplectization, Reeb flow
   (f) Darboux' theorem for contact manifolds, Gray's theorem
   (g) integrable vs. non-integrable complex structures
   (h) Kähler manifolds: definition, examples, Kähler forms as $(1,1)$-forms
3. J-holomorphic curves
   (a) definition, equivalent forms of the defining equations, energy
   (b) linearization of the $\overline{\partial}_J$ operator and its role in the theory
   (c) Cauchy-Riemann operators
   (d) statement and consequences of the Carleman similarity principle
   (e) moduli spaces of J-holomorphic curves, strategy of proof for regularity
   (f) compactness for moduli spaces of J-holomorphic curves, basic phenomena, including bubbling
   (g) statement and proof of the nonsqueezing theorem

Advice for exam preparation

Of course you will need to know the statements and proofs of the main results. Make sure you also know and understand *many* examples. For instance, in the exam I might ask you to write down a function on $\mathbb{R}^{2n}$ whose Hamiltonian flow will have a prescribed effect, or to compute the Maslov index of some explicit loop of Lagrangian subspaces, or to give an example of a J-holomorphic curve with certain properties, or....

Other questions I like to ask include: What happens to a given theorem when you leave out one of the assumptions? Do you know counterexamples? What is a simple situation where a given theorem is useful?

As you prepare for the exam, look back at the exercises, as they often give a valuable second perspective on topics covered in the lecture.

You will be allowed to start the exam with a topic of your choice.