Universität Hamburg
Janko Latschev
Pavel Hajek

Winter 2019/20

# Symplectic Geometry

## Problem Set 1

**1.** Prove that a 2-form $\omega$ on a $2n$-dimensional real vector space $V$ is symplectic if and only if

$$\omega^n = \omega \wedge \cdots \wedge \omega \neq 0.$$

**2.** Recall that for a linear subspace $W$ of a symplectic vector space $(V, \omega)$ we defined the $\omega$-orthogonal complement as

$$W^{\perp_\omega} := \{v \in V \,:\, \omega(w, v) = 0 \text{ for all } w \in W\}.$$

**a)** Prove that $\dim W + \dim W^{\perp_\omega} = \dim V$!

**b)** Prove that $(W^{\perp_\omega})^{\perp_\omega} = W$!

**c)** Prove that $W$ is a symplectic subspace if and only if $(W, \omega|_W)$ is a symplectic vector space if and only if $V = W \oplus W^{\perp_\omega}$!

**d)** More generally, prove that the quotient space $W/(W \cap W^{\perp})$ always inherits a symplectic structure from $V$.

**3.** Prove that a linear subspace $W$ of codimension 1 in a symplectic vector space $(V, \omega)$ is always coisotropic!

**4.** Prove that the standard euclidean scalar product $\langle\,.\,,\,.\,\rangle_{\mathrm{st}}$, the standard complex structure $J_{\mathrm{st}}$ and the standard symplectic form $\omega_{\mathrm{st}}$ on $\mathbb{R}^{2n}$ are related by

$$\langle u, v\rangle_{\mathrm{st}} = \omega_{\mathrm{st}}(u, J_{\mathrm{st}} v).$$

**5.** Prove that $\mathrm{Sp}(2, \mathbb{R})$ is diffeomorphic to the open solid torus $S^1 \times \mathbb{R}^2$!

**6.** Give examples of elements of $\mathrm{SL}(4, \mathbb{R})$ which are not elements of $\mathrm{Sp}(4, \mathbb{R})$!