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Differential Topology

Problem Set 4

1. Recall that two continuous maps f_0 and f_1 between topological spaces X and Y are called *homotopic* if there exists a continuous map $F : [0,1] \times X \to Y$ such that $F(0,x) = f_0(x)$ and $F(1,x) = f_1(x)$.

a) Show that if M is a C^1 manifold of dimension $n < m$, then any C^1 map $f : M \to S^m$ is C^1 homotopic to a constant map.

b) Let M and N be two manifolds of class C^r and let f_0 and f_1 be two C^r maps between them which are continuously homotopic. Prove that the two maps are also C^r homotopic.

c) Prove that for any $r \geq 0$ homotopy classes of maps from M to N are open in $C^r(M,N)$ with the strong topology.

d) Use this to prove that the statement in part **a)** is also true for continuous maps from M to S^m, as long that M has a differentiable structure.

2. Prove that if M is a connected manifold of class C^r with $r \geq 0$, then any two distinct points $x \neq y$ in M can be connected by an embedded C^r path, i.e. there exists an embedding $\gamma : [0,1] \to M$ of class C^r such that $\gamma(0) = x$ and $\gamma(1) = y$.

3. Prove that every compact connected 1-dimensional manifold without boundary is homeomorphic to S^1. Analogously, every compact connected 1-dimensional manifold with non-empty boundary is homeomorphic to the interval $[0,1]$.