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Symplectic Homology

Problem Set 5

I realize this is too much for a week, but do try to think about all problems, then pick what you like most to work on seriously.

1. Let Σ be a compact surface with one boundary component. Prove that there exists a Liouville structure on Σ, i.e. a 1-form λ such that $d\lambda$ is a volume form and such that the Liouville vector field associated to λ points outward along the boundary!
Hints: Since any Liouville form can be scaled, you may arrange that the total volume of $(\Sigma, d\lambda)$ *is equal to 1. What should be the integral of* λ *over* $\partial\Sigma$ *in this case? First construct a form* λ_1 *with support near the boundary of* Σ *which has the correct boundary integral and such that* $d\lambda_1 \neq 0$ *near* $\partial\Sigma$*. Now construct a volume form* ω *on* Σ *of total volume 1 which agrees with* $d\lambda_1$ *near the boundary (this is easy). Finally apply de Rham's theorem (that integration gives an isomorphism from compactly supported de Rham cohomology of* $\Sigma \setminus \partial\Sigma$ *to* $\mathbb{R}$*) to the difference* $\omega - d\lambda_1$ *to complete the proof.*

2. **a)** Generalize the previous proof to show that given a connected subdomain with boundary $\Sigma_0 \subset \Sigma$ inside a surface with boundary, one can always construct a Liouville structure on Σ such that $\Sigma_0 \subset \Sigma$ is an exactly embedded Liouville subdomain!

b) One can even prescribe the volumes to be any given values $0 < \mathrm{vol}(\Sigma_0) < \mathrm{vol}(\Sigma)$. Given these volumes, give an estimate on the maximal possible size of the piece of the positive half of the symplectization of $\partial\Sigma_0$ which can be embedded as a collar neighborhood of $\partial\Sigma_0$ into $\Sigma \setminus \mathrm{Int}(\Sigma_0)$!

3. Compute the symplectic homology of the surface with boundary obtained by deleting an open disk from the 2-dimensional torus!
Hint: You don't have to actually calculate any holomorphic curves, but you do need a bit of topology.

4. Explain as precisely as you can what goes wrong in our construction of the Viterbo transfer map if the Liouville embedding is not exact!

Bitte wenden!

5. Study Weinstein handle attachment, either from Weinstein's original paper or from some book of your choice (part of the exercise is to find one that explains it). Prepare an explanation for your fellow students!

6. Study section 2 of the paper "Applications of symplectic homology II: Stability of the action spectrum" by Cieliebak, Floer, Hofer and Wysocki (see the homepage of the course for a link) to understand how one actually can construct the time-dependent perturbation of the Hamiltonians I have been referring to in class, and prepare an explanation for your fellow students!