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Symplectic Homology

Problem Set 4

1. Express the conditions for a compatible almost complex structure on the symplectzation of a contact manifold (V, α) to be convex in the coordinates where the symplectization is given by $(\mathbb{R} \times V, d(e^s\alpha))$!

2. We view $\mathbb{C}$ as the completion of the ball $\overline{B(0,1)}$, viewed as a Liouville domain with Liouville form $\lambda = \frac{1}{2}(x\,dy - y\,dx)$, and consider the family of Hamiltonians $H_b : \mathbb{C} \to \mathbb{C}$ given as

$$H_b(z) = b|z|^2 - b.$$

a) Prove that these Hamiltonians are linear with slope b according to our definitions (this involves writing out the explicit parametrization of the cylindrical end)!

b) What conditions do we have to impose on b in order for H_b to have only nondegenerate 1-periodic orbits? How many 1-periodic orbits are there for such an admissible $b \in (0, \infty)$?

c) Compute $HF_*(H_b, J_0)$ for such Hamiltonians (where J_0 is the standard almost complex structure, i.e. multiplication by i)! Pay special attention to gradings!

d) How much of this discussion generalizes to $\mathbb{C}^n$?

3. **a)** Prove that for any smooth closed manifold Q the first Chern class of its cotangent bundle $(T^*Q, \omega_{\text{can}})$ vanishes!
Hint: Write $T^*Q|_Q$ as the complexification of a real vector bundle, and then check what the literature says about Chern classes of such bundles.

b) Prove that for any regular level set $f^{-1}(c) \subset \mathbb{C}^N$ of a holomorphic function $f : \mathbb{C}^N \to \mathbb{C}$ the first Chern class vanishes! How does this generalize to complex submanifolds of higher codimension?

Bitte wenden!

4. Let (W, λ) be a Liouville domain and let $(\widehat{W}, \widehat{\lambda})$ be its completion. Suppose $H^{\pm} : \widehat{W} \to \mathbb{R}$ are linear Hamiltonians with slopes $b^- \geq b^+$ on $[r_0, \infty) \times (\partial W)$, where $b^{\pm}$ are both not in the action spectrum of $\lambda|_{\partial W}$. Prove that the image of a continuation cylinder as defined in the lecture (in particular, $H_{s,t}$ has slope $b(s)$ on $[r_0, \infty) \times (\partial W)$ with $b'(s) \leq 0$) cannot intersect $[r_0, \infty) \times (\partial W)$, by adapting the proof given in the lecture for Floer cylinders of a single linear Hamiltonian!