Introduction to symplectic geometry

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Summary of topics

1. Linear symplectic geometry
   (a) the different types of linear subspaces and their normal forms
   (b) linear symplectic maps and their properties
   (c) relations between $\mathrm{Sp}(2n)$, $U(n)$ and $O(2n)$
   (d) Maslov indices for loops of symplectic matrices and of Langrangian subspaces: definitions, properties and computations in examples
   (e) compatible complex structures, contractibility of $\mathcal{J}(\omega_{\mathrm{st}})$
   (f) relation to hermitian metrics
2. Basic on symplectic manifolds
   (a) basic definitions and examples
   (b) symplectic and Hamiltonian diffeomorphisms
   (c) Moser's argument with applications, especially Darboux' theorem
   (d) special submanifolds, examples of normal forms for neighborhoods
   (e) contact manifolds, symplectization, Reeb flow
   (f) Darboux' theorem for contact manifolds, Gray's theorem
   (g) integrable vs. non-integrable complex structures
   (h) Kähler manifolds: definition, examples, Kähler forms as $(1,1)$-forms
3. Symplectic capacities
   (a) definition, relation to nonsqueezing
   (b) Gromov width: definition and computation in examples
   (c) construction of the Hofer-Zehnder capacity (ingredients, strategy and key steps of the proof of existence)
   (d) applications of capacities: nonsqueezing, special cases of Weinstein conjecture, etc.

Advice for exam preparation

Of course you will need to know the statements and proofs of the main results. Make sure you also know and understand *many* examples. For instance, in the exam I might ask you to write down a function on $\mathbb{R}^{2n}$ whose Hamiltonian flow will have a prescribed effect, or to compute the Maslov index of some explicit loop of Lagrangian subspaces.

Other questions I like to ask include: What happens to a given theorem when you leave out one of the assumptions? Do you know counterexamples? What is a simple situation where a given theorem is useful? How is it proven?

As you prepare for the exam, look back at the exercises, as they often give a valuable second perspective on topics covered in the lecture.

Exam dates

The oral exam will take between 20 and 30 minutes, and you can take it in either on February 12/13 or in the week starting March 23. Please tell me your preferred date before January 22. I will announce the exam dates in the last week of classes.