Homework 8, due Monday 11 April

1. Show that in the Rieger-Nishimura ladder, $w_i \models g_n(p)$ iff $w_n R w_i$. You can assume that the cases $n = 0, 1, 2$ are true. [5 pts]

2. (a) Let $\mathfrak{M}$ and $\mathfrak{N}$ be two **IPC**-models, and f a **frame**-p-morphism from $\mathfrak{M}$ to $\mathfrak{N}$. Let $\psi_1, \ldots, \psi_n$ be such that for each $w \in W$ and each $i \leq n$, $w \models \psi_i$ iff $f(w) \models p_i$. Prove that for each $\varphi(p_1, \ldots, p_n)$ we have $w \models \varphi(\psi_1, \ldots, \psi_n)$ iff $f(w) \models \varphi(p_1, \ldots, p_n)$. [4 pts]

   (b) Assume that $\mathfrak{M}$ is a model with a root w_0 such that $w_0 \models \neg\neg\psi$ and $w_0 \not\models \psi$. Define a **frame**-p-morphism f from $\mathfrak{M}$ to $\mathcal{RN}_{w_2}$ in such a way that, for each $w \in W$, $w \models \psi$ iff $f(w) \models p$. [3 pts]

3. Assume $\mathfrak{M} \models \neg\neg\varphi$ and $\mathfrak{M} \not\models \varphi$. Assume that $\mathfrak{N} \models \neg\varphi$. Now add nodes w'_n for $n \geq i$ to the union of $\mathfrak{M}$ and $\mathfrak{N}$ to obtain a new model $\mathfrak{M}^*$. Describe the accessibility relation of the new model.

   With the help of the p-morphism of exercise 2 (b), construct a p-morphism from $\mathfrak{M}^*$ to the Rieger-Nishimura ladder. Then use exercise 2 (a) to show that $\mathfrak{M}^* \not\models g_n(\varphi)$ for each n.

   Conclude theorem 40. [6 pts]