ZFC^* = some finite fragment of ZFC.

Reflection: For any finite $\phi_1 \dots \phi_n$ there is M (of the right type) such that

$$(\phi_1^M \leftrightarrow \phi_1) \wedge (\phi_2^M \leftrightarrow \phi_2) \wedge \dots \wedge (\phi_n^M \leftrightarrow \phi_n)$$

Corollary: For any finite $ZFC^* \subseteq ZFC$, there is M (of right type) s.t. $M \models ZFC^*$

In practice: ZFC^*

(ZFC_n - restricting Compr. & Repl. to formulas of Σ_n-complexity.)

$p \Vdash^* \varphi \iff$ .... induction on names & formulas.

(1) $p \Vdash^* \tau = \sigma \iff \forall \langle \pi, q \rangle \in \tau$ .... $q \Vdash^*$ .....

rec. on w.f.d.d.-relation.

lower rank

lower rank.

(2) $p \Vdash^* \tau \in \sigma \iff$ ...

(3) $\boxed{p \Vdash^* \varphi}$

meta-induction. For any φ in $\mathcal{L}_\in$, $\boxed{p \Vdash^* \varphi}$

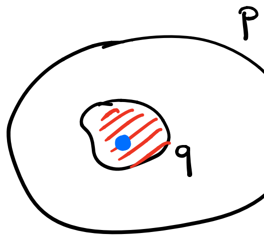
(NB: take $\mathbb{P}$ = trivial, $\mathbb{1} \in \mathbb{P}$, and $M[G] = M$

Then $p \Vdash^* \varphi \leftrightarrow \varphi$

$p \geqslant q$ inverted (Shelah)

$\uparrow$ stronger

$P(x)$:



$q \leq p$

$q \subseteq p$

q str. than p

For many $\mathbb{P}$, $p \subseteq \omega^\omega$ and $p \Vdash \dot{x}_G \in$ "p"