Homework Week 9
Due 7 January 2019

1. (a) Write down the $\mathbb{P}$-name $\check{3}$ in detail. [1 point]

   (b) Let $\tau = \{(\varnothing, p), (\{(\varnothing, q)\}, r)\}$. Compute τ_G for each of the 8 possibilities for p, q, r being $\in G$ or $\notin G$. [2 points]

2. In the lecture we saw that if $\mathbb{P}$ is non-atomic, then $\mathbb{P}$-generic filters over M are not in M. Prove the converse: if $\mathbb{P}$ is atomic (i.e., not non-atomic), then there exists a $G \in M$ which is a $\mathbb{P}$-generic filter over M. [2 points]

   [*Hint: let* $G = \{p \ : \ p \parallel r\}$ *for suitable* r.]

3. A subset $D \subseteq \mathbb{P}$ is called *dense below* p if $\forall q \leq p \, \exists r \leq q \, (r \in D)$.

   (a) Let $D \subseteq \mathbb{P}$. Show that, if $\{q : D \text{ is dense below } q\}$ is dense below p, then D is dense below p. [1 point]

   (b) Show that if $D \in M$ is dense below p and G is $\mathbb{P}$-generic over M, then

   $$p \in G \rightarrow G \cap D \neq \varnothing$$ [2 points]

4. Let $\mathbb{P}$ be non-atomic and let M be a countable transitive model. Prove that

   $$|\{G : G \text{ is a } \mathbb{P}\text{-generic filter over } M\}| = 2^{\aleph_0}.$$ [2 points]