Homework Week 5
Due 26 November 2018

1. Let $M \preccurlyeq H_{\omega_2}$ be a *countable* elementary submodel. Note that M is *not transitive*.

   (a) Give an example of a set x such that $x \in M$ but $x \not\subseteq M$, and a set y such that $y \subseteq M$ but $y \notin M$. [2 points]

   (*Hint: use previous homework.*)

   (b) Suppose $y \subseteq M$ and y is finite. Then $y \in M$. [2 points]

   (c) Suppose $x \in M$ and x is countable. Then $x \subseteq M$. [2 points]

   (*Hint: $H_{\omega_2} \models$ there is a surjection from ω to x. Another hint: $\omega \in M$.*)

2. The *height* of a set-model M of set theory, $o(M)$, is defined as $\mathrm{Ord} \cap M$. Show that:

   (a) If M is transitive, then $o(M)$ is an ordinal, it is the least ordinal not in M, and it is a limit ordinal. [1 point]

   (b) Give an example of a non-transitive set-model $M \models (\mathsf{ZFC} \setminus \textsc{Power Set})$ such that $o(M)$ is not an ordinal. [1 point]

   (*Hint: use Exercise 1*)

   (c) Prove that if $M \preccurlyeq H_{\omega_2}$ is countable, then $M \cap \omega_1$ is a countable ordinal. [2 points]

   (*Hint: use Exercise 1*)