Homework Week 11
Due 28 January 2019

1. This is just to repeat some basic facts about "new" vs. "old" objects and to practice dealing with dense sets which are in M or not in M.

   (a) Let M be a model of ZFC, let $\mathbb{C} = \omega^{<\omega}$ be Cohen forcing and G a $\mathbb{C}$-generic filter over M. Let $f_G = \bigcup G$. Show that $f_G \in \omega^\omega \setminus M$. [1 point]

   (b) What's wrong with the following argument: "Let $f_G \in \omega^\omega$ be as above. Define $D := \{p \in \mathbb{C} \,:\, p(n) \neq f_G(n) \text{ for some } n\}$. Clearly this is a dense subset of $\mathbb{C}$. Therefore $G \cap D \neq \varnothing$. Therefore for some n we have $f_G(n) \neq f_G(n)$ — contradiction!" [1 point]

2. Let $\mathbb{C} = \omega^{<\omega}$ be Cohen forcing and $M[G]$ a $\mathbb{C}$-generic extension over M. Show that $f_G := \bigcup G$ has the following property: for every $x \in M$, there are infinitely many $n \in \omega$, such that $x(n) < f_G(n)$ (we say that f_G *is an unbounded real over* M). [4 points]

   Hint: for every $x \in \omega^\omega \cap M$ and every $k \in \omega$, define appropriate dense sets

   $$D_{x,k} = \{p \in \mathbb{C} \,:\, \exists n \geq k \ldots\ldots\}$$

3. Let $a, S \subseteq \omega$ be infinite sets. We say that S *splits* a if both $a \cap S$ and $a \setminus S$ are infinite (so S splits a into two infinite parts). If $M \subseteq M[G]$ is a generic extension and $S \in M[G]$, then we say that S *is a splitting real over* M, if for every $a \in [\omega]^\omega \cap M$, S splits a. Clearly, a *splitting real* S cannot be in M. Show that if f_G is as above, then $\{n \,:\, f_G(n) = 0\}$ is a splitting real over M. [4 points]

   Hint: for every infinite $a \subseteq \omega$ and every $k \in \omega$, define appropriate dense sets $D_{a,k}$.