1. This is just to repeat some basic facts about “new” vs. “old” objects and to practice dealing with dense sets which are in $M$ or not in $M$.

   (a) Let $M$ be a model of ZFC, let $\mathbb{C} = \omega^{<\omega}$ be Cohen forcing and $G$ a $\mathbb{C}$-generic filter over $M$. Let $f_G = \bigcup G$. Show that $f_G \in \omega^\omega \setminus M$. \hfill [1 point]

   (b) What’s wrong with the following argument: “Let $f_G \in \omega^\omega$ be as above. Define $D := \{p \in \mathbb{C} : p(n) \neq f_G(n) \text{ for some } n\}$. Clearly this is a dense subset of $\mathbb{C}$. Therefore $G \cap D \neq \emptyset$. Therefore for some $n$ we have $f_G(n) \neq f_G(n)$ — contradiction!” \hfill [1 point]

2. Let $\mathbb{C} = \omega^{<\omega}$ be Cohen forcing and $M[G]$ a $\mathbb{C}$-generic extension over $M$. Show that $f_G := \bigcup G$ has the following property: for every $x \in M$, there are infinitely many $n \in \omega$, such that $x(n) < f_G(n)$ (we say that $f_G$ is an unbounded real over $M$). \hfill [4 points]

   Hint: for every $x \in \omega^\omega \cap M$ and every $k \in \omega$, define appropriate dense sets

   $$D_{x,k} = \{p \in \mathbb{C} : \exists n \geq k \ldots \}$$

3. Let $a, S \subseteq \omega$ be infinite sets. We say that $S$ splits $a$ if both $a \cap S$ and $a \setminus S$ are infinite (so $S$ splits $a$ into two infinite parts). If $M \subseteq M[G]$ is a generic extension and $S \in M[G]$, then we say that $S$ is a splitting real over $M$, if for every $a \in [\omega]^\omega \cap M$, $S$ splits $a$. Clearly, a splitting real $S$ cannot be in $M$. Show that if $f_G$ is as above, then $\{n : f_G(n) = 0\}$ is a splitting real over $M$. \hfill [4 points]

   Hint: for every infinite $a \subseteq \omega$ and every $k \in \omega$, define appropriate dense sets $D_{a,k}$. 

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**Homework Week 11**  
**Due 28 January 2019**