Homework Week 1

Due on 22 October 2018

1. For each of the following statements, determine whether they are made in the formal language of set theory, or in a meta-language.[1] [3 points]

   (a) Every convergent sequence in $\mathbb{R}$ is bounded from above.

   (b) ZFC $\vdash$ " Every convergent sequence in $\mathbb{R}$ is bounded from above".

   (c) All dense countable linear order without end-points are isomorphic to each other.

   (d) ZFC is consistent.

   (e) The language of set theory consists of one binary relation symbol $\in$.

   (f) If ZFC is consistent, then CH is not a theorem of ZFC.

   (g) $\forall \alpha \in ON\ (\alpha = \varnothing)$.

   (h) ZFC $\nvdash \forall \alpha \in ON\ (\alpha = \varnothing)$.

   (i) ZFC contains infinitely many axioms.

   (j) "$\forall x \forall y\ (x = y)$" is not an axiom of ZFC.

   (k) The addition operation on the ordinals is not commutative.

   (l) Ord (the class of all ordinals) is not a set.

   (m) There are classes which are not sets.

2. Consider the following informally stated assertion:

   "For every proper class A and every set X, there exists an injective function $f : X \to A$."

   (a) Write down the above statement formally. You may use the abbreviations "f is a function", "dom(f)" and "ran(f)" without writing them out in detail. [3 points]

   (b) Is this a statement in the formal language or the meta-language? [1 point]

   (c) Prove the above assertion (using an informal argument which is, in principle, formalizable in ZFC). [3 points]

[1] As mentioned, any statement in the meta-language can also be viewed as a statement in the formal language, so this exercise is about the ordinary/most obvious meaning.