1. For each of the following statements, determine whether they are made in the formal language of set theory, or in a meta-language.\(^1\) [3 points]

(a) Every convergent sequence in \(\mathbb{R}\) is bounded from above.
(b) \(\text{ZFC} \vdash \text{"Every convergent sequence in } \mathbb{R} \text{ is bounded from above".}\)
(c) All dense countable linear order without end-points are isomorphic to each other.
(d) \(\text{ZFC}\) is consistent.
(e) The language of set theory consists of one binary relation symbol \(\in\).
(f) If \(\text{ZFC}\) is consistent, then \(\text{CH}\) is not a theorem of \(\text{ZFC}\).
(g) \(\forall \alpha \in \text{ON} (\alpha = \emptyset)\).
(h) \(\text{ZFC} \not\vdash \forall \alpha \in \text{ON} (\alpha = \emptyset)\).
(i) \(\text{ZFC}\) contains infinitely many axioms.
(j) \(\forall x \forall y (x = y)\) is not an axiom of \(\text{ZFC}\).
(k) The addition operation on the ordinals is not commutative.
(l) \(\text{Ord}\) (the class of all ordinals) is not a set.
(m) There are classes which are not sets.

2. Consider the following informally stated assertion:

“For every proper class \(A\) and every set \(X\), there exists an injective function \(f : X \to A\).”

(a) Write down the above statement formally. You may use the abbreviations “\(f\) is a function”, “\(\text{dom}(f)\)” and “\(\text{ran}(f)\)” without writing them out in detail. [3 points]
(b) Is this a statement in the formal language or the meta-language? [1 point]
(c) Prove the above assertion (using an informal argument which is, in principle, formalizalizable in \(\text{ZFC}\)). [3 points]

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\(^1\)As mentioned, any statement in the meta-language can also be viewed as a statement in the formal language, so this exercise is about the ordinary/most obvious meaning.