Practice exercises for Monday 7 January

1. In the following, σ, τ, θ are $\mathbb{P}$-names in M and G is a $\mathbb{P}$-generic filter over M. Are the following true or false?
   (a) If $(\sigma, \mathbf{1}) \in \tau$ then $\sigma_G \in \tau_G$.
   (b) If $(\sigma, p) \in \tau$ and $p \in G$, then $\sigma_G \in \tau_G$.
   (c) If $\sigma_G \in \tau_G$ then $(\sigma, \mathbf{1}) \in \tau$.
   (d) If $x \in \tau_G$ then there exists $(\sigma, p) \in \tau$ such that $p \in G$ and $x = \sigma_G$.
   (e) If $\sigma_G \in \tau_G$ then there exists $p \in G$ such that $(\sigma, p) \in \tau$.
   (f) If $\sigma_G \in \tau_G$ then there exists $(\theta, r) \in \tau$ such that $r \in G$ and $\theta_G = \sigma_G$.

2. Let σ, τ be two $\mathbb{P}$-names in M and let G be generic over M. Show that $(\sigma \cup \tau)_G = \sigma_G \cup \tau_G$

3. If σ, τ are two $\mathbb{P}$-names, let $up(\sigma, \tau) = \{(\sigma, \mathbf{1}), (\tau, \mathbf{1})\}$ and $op(\sigma, \tau) = up(up(\sigma, \sigma), up(\sigma, \tau))$. Show that $up(\sigma, \tau)_G = \{\sigma_G, \tau_G\}$ and $op(\sigma, \tau)_G = \langle \sigma_G, \tau_G \rangle$.