Practice exercises for Monday 3 December

1. A theory T is *complete* if for all φ, either $T \vdash \varphi$ or $T \vdash \neg\varphi$. For a model $\mathcal{A}$, the *theory of* $\mathcal{A}$, denoted by $Th(\mathcal{A})$, is the collection $\{\varphi : \mathcal{A} \models \varphi\}$. Are the following true or false? Give an argument or a counterexample.

   (a) For every $\mathcal{A}$, $Th(\mathcal{A})$ is consistent and complete.

   (b) If T is consistent and complete, then, up to isomorphisms, there is exactly one model $\mathcal{A}$ such that $\mathcal{A} \models T$.

   (c) For any model $\mathcal{A}$, if $\mathcal{B} \models Th(\mathcal{A})$ then $\mathcal{A} \cong \mathcal{B}$.

   (d) For any finite model $\mathcal{A}$, if $\mathcal{B} \models Th(\mathcal{A})$ then $\mathcal{A} \cong \mathcal{B}$.

   (e) If T is consistent, complete, and "contains witnesses" (*enthält Beispiele*)—i.e., if $T \vdash \exists x \phi(x)$ then there is a constant $\dot{c}$ such that $T \vdash \phi(\dot{c})$—then there exists an $\mathcal{A}$ such that $T = Th(\mathcal{A})$.

2. Prove that $V = L$ implies that $L_\kappa = H_\kappa$ for all $\kappa \geq \omega$.