1. Let $D = (V, A)$ be a digraph with $f \leq g \in \mathbb{R}^A$. Let $m \in \mathbb{R}^V$ be given such that $\sum_{v \in V} m(v) = 0$. Give a necessary and sufficient condition for the existence of an $x \in \mathbb{R}^A$ with $f \leq x \leq g$ and $\varrho_x(v) - \delta_x(v) = m(v)$ for $v \in V$.

2. Let $G = (V, E)$ be a graph and $f \in \mathbb{N}^V$. Let us denote the number of edges with at least one endpoint in $X \subseteq V$ by $e(X)$. We want to orient the edges of G in such a way that the indegree of v is at least $f(v)$ for each $v \in V$. Check that the condition, $\sum_{v \in X} f(v) \leq e(X)$ for every $X \subseteq V$, is necessary. Prove that it is actually sufficient as well. (Hint: pick and arbitrary orientation D first and define a suitable polyhedron using the incidence matrix of D.)

3. Consider the previous problem but with upper bounds $g \in \mathbb{N}^E$. Give a condition which is obviously necessary for the existence of an orientation with $\varrho(v) \leq g(v)$ for each v. Is it sufficient?

4. Let $G = (V, E)$ be a graph and $f \leq g \in \mathbb{N}^V$. Assume that G has an orientation D_1 for which $f(v) \leq \varrho_{D_1}(v)$ for $v \in V$ and an orientation D_2 for which $\varrho_{D_2}(v) \leq g(v)$ for $v \in V$. Show that there is an orientation D_3 of G for which $f(v) \leq \varrho_{D_3}(v) \leq g(v)$ for $v \in V$.

5. Let G be a 2-edge-connected graph. Show that it has a strongly connected orientation in which $|\varrho(v) - \delta(v)| \leq 1$ for each vertex. (Hint: ear decomposition)

6. Suppose that D and D' are strongly connected orientations of the graph G. Show that there is a finite sequence $D = D_0, D_1, \ldots, D_k = D'$ of strongly connected orientations of G in which one can get D_{i+1} by reversing a directed cycle or directed path of D_i.