1. Give a polyhedron $P \subseteq \mathbb{R}^n$ for which $\mathsf{conv}(P \cap \mathbb{Z}^n)$ is not a polyhedron, where **conv** stands for convex hull. (Hint: if P is representable by an integral linear inequality system, then $\mathsf{conv}(P \cap \mathbb{Z}^n)$ is a polyhedron.)

2. Show that every circulation z is a linear combination of characteristic vectors of directed cycles.

3. Let D be a strongly connected **tournament** (a digaph that we obtain by orienting a complete graph). Prove that it has a Hamiltonian cycle.

4. The edges of a tournament are coloured with red and blue. Show that there is a vertex from which every vertex is reachable by a monochromatic path.

5. Let $D = (V, A)$ be strongly connected. Show that there is an $F \subseteq A$ such that F meets with all the directed cycles of D and for every $e \in A$ there is directed cycle C through e with $|F \cap C| = 1$.

Hint:

1. Show first that whenever the desired F exists it can be chosen in such a way that from a given vertex t every vertex is reachable without using edges from F.
2. Strongly connected digraphs have ear decomposition. Apply induction on the number of ears.
3. Show the easy case when the last ear is a cycle.
4. Let the last ear be an $s \to t$ path P. Combine the induction hypothesis and hint 1.

6. Let $A \in \mathbb{Z}^{n \times n}$ be regular and $b \in \mathbb{Z}^n$. Prove that $\nexists x \in \mathbb{Z}^n$ such that $Ax = b$ iff $\exists y \in \mathbb{Q}^n$ such that yA is integral but yb is not.