1. Give the dual of the following system.

$$\max c_0x_0 + c_1x_1$$
$$x_0 \geq \underline{0}$$
$$Ax_0 + Bx_1 = b_0$$
$$Cx_0 + Dx_1 \leq b_1$$

2. Show that in a bipartite graph without isolated vertices the maximal number of independent vertices equals to the minimal number of edges covering all the vertices.

3. Derive Hall's Theorem (there is no matching covering S in the bipartite graph $G = (S, T, E)$ iff for some $X \subseteq S$ there is less than $|X|$ vertices in T adjacent with a vertex in X) with polyhedral tools.

4. Let $D = (V, A)$ be a digraph with $s \neq t \in V$. Suppose that s has no ingoing and t has no outgoing edges. An $s \to t$ **flow** is an $x : A \to \mathbb{R}^+$ such that $\varrho_x(v) = \delta_x(v)$ (here $\varrho_x(v)$ is the sum of the values of x on the ingoing edges of v and $\delta_x(v)$ is the same for outgoing edges) for $v \in V \setminus \{s, t\}$. The **amount** of the flow is $\delta_x(s)$.

Show that $\delta_x(s) = \delta_x(S) - \varrho_x(S)$ whenever $s \in S \subseteq V - t$.

Let a capacity $g : A \to \mathbb{R}^+$ be given. The flow x is called **feasible** if $x \leq g$. Write a linear program about finding a feasible flow of maximal amount and show that the value of the optimum is the following:

$$\min\{\varrho_g(U) \,:\, U \subseteq V,\ t \in U,\ s \notin U\} =: \boldsymbol{\lambda_g(s,t)}.$$

5. Let $D = (V, A)$ be a digraph. A $\Delta \in \mathbb{R}^A$ is a **tension** if there is some $\pi \in \mathbb{R}^V$ for which $\Delta(uv) = \pi(v) - \pi(u)$ for $uv \in A$. Check that tensions form a linear subspace of $\mathbb{R}^A$ and characterise the orthogonal complement of this subspace.

6. Let $D = (V, A)$ be a digraph and let Q be the incidence matrix of it. Prove that the underlying undirected graph is a forest iff the columns of Q are linearly independent.

7. Let $\mathcal{H}$ be a laminar hypergraph (i.e. two hyperedges are either disjoint or one of them is a subset of the other). Show that the incidence matrix of $\mathcal{H}$ is TU.