1. Give a sequence (P_n) of nonempty polyhedra such that $P_n \subseteq \mathbb{R}^n \setminus \{\underline{0}\}$ and it has no proper face.

2. Give a sequence (P_n) of polyhedra such that P_n has at least 2^n vertices but at most $O(n)$ maximal proper faces.

3. Let Q be a regular TU matrix and let b be integer-valued. Show that the unique solution of $Qx = b$ is integer-valued. (Hint: use Cramer's rule).

4. The vertices u, v of P are **neighbours** if the inner points of the segment $[u, v]$ have rank 1. Characterise this relation between u and v using $Q_u^{=}$ and $Q_v^{=}$.

5.* Let $D = (V, A)$ be a digraph and let $R_i \subseteq V$ be nonempty for $i = 1, \ldots, k$. Assume that for every $v \in V$ there is a system $\{P_i^v : 1 \leq i \leq k\}$ of edge-disjoint paths such that P_i^v goes from R_i to v. Show that one can colour the edges with the indices $1, \ldots, k$ such that for every i all vertices are reachable from R_i using only edges of colour i. Hints:

1. Check that the condition is equivalent with the following: every nonempty $X \subseteq V$ has at least as many ingoing edges as many R_i are disjoint from it.
2. It is enough to show that if $R_i \neq V$ for some i, then there is an outgoing edge e of R_i such that if we delete e and extend R_i with $\mathsf{head}(e)$, then the condition remains true for the resulting system.
3. Prove that $e \in \mathsf{out}(R_i)$ is a "bad" choice iff it enters a set X such that $X \cap R_i \neq \varnothing$ and X has as many ingoing edges as many R_j are disjoint from it (i-dangerous set).
4. If X, Y are i-dangerous then so does $X \cap Y$ unless it is empty.
5. Show that a $\subseteq$-minimal i-dangerous set Z spans an outgoing edge of R_i.

5. A digraph $D = (V + r, A)$ is **rooted k-edge-connected** from r if every nonempty vertex set $X \not\ni r$ has at least k ingoing edges. Prove that rooted k-edge-connectedness is equivalent with having k many pairwise edge-disjoint spanning r-arborescences.