**1.** Formulate

$$\max_x \min_{1\le i\le n} \{x_i : Qx \le b, x \ge \underline{0}\}$$

as a linear programming problem.

**2.** If $z$ is in the convex hull of $x_1, \ldots, x_m \in \mathbb{R}^n$, then there are at most $n+1$ among the $x_i$ with this property.

**3.** Let $G = (V, E)$ be a finite graph which has a perfect matching. For a perfect matching $M$, let $x_M : E \to \{0, 1\}$ be its characteristic vector (i.e. $x_M(e) = 1$ iff $e \in M$). Write linear inequalities that are valid for all these $x_M$ vectors.

**4.** Let $D = (V, A)$ be a finite digraph with $r \in V$. Suppose that every vertex is reachable from $r$. An $F \subseteq A$ is a spanning ***r*-arborescence** if it is a tree in undirected sense and every vertex in $V$ is reachable from $r$ in $F$. Let $x_F$ be the characteristic vector of $F$. Write linear inequalities that are valid for all these $x_F$ vectors.

**5.** Let $\mathcal{L}$ be a linear subspace of $\mathbb{R}^n$. Show that exactly one of $\mathcal{L}, \mathcal{L}^\perp$ contains an $x \ge \underline{0}$ with $x_n > 0$.

**6.** Is it true that there always exists an $x \in \mathcal{L}$ and $y \in \mathcal{L}^\perp$ such that $x + y$ has only positive coordinates?

**7.** Let $\mathcal{M}$ be the set of those polyhedra $P$ that can be represented in the form $Ax \le \underline{1},\ x \ge \underline{0}$, where $A$ has only non-negative components and in every column there is at least one strictly positive ($\underline{1}$ is the constant 1 vector). Let

$$f(P) = \{y \ge \underline{0} \,:\, \forall x \in P\ yx \le 1\}.$$

Prove that $f$ is an $\mathcal{M}$-invariant involution.