

1. Let $P := \{x \in \mathbb{R}^3 : 0 \leq x_i \leq 1, i = 1, 2, 3\}$. What are the basic solutions of P ? What are the solutions of rank 1?
2. Show a polyhedron that has infinitely many basic solutions.
3. For a generated cone K let $K^* := \{x : \forall y \in K \ x y \leq 0\}$ (the **polar** of the cone). Show that $K^{**} = K$.
4. Let $P \subseteq \mathbb{R}^n$ be a polyhedron. For $z \in P$ let $r(z)$ be the rank of z in P . What can be the range of the function $r(x)$?
5. The inequality $ax \leq \alpha$ is a **logical consequence** of the inequality system $Qx \leq b$ if

$$\forall x (Qx \leq b \implies ax \leq \alpha).$$

If there is some $y \geq \underline{0}$ such that $yQ = a$ and $yb \leq \alpha$, then $ax \leq \alpha$ is a **linear consequence** of the inequality system $Qx \leq b$. Prove the logical and linear consequences are the same.