1. Let $G=(V,E)$ be a $2k$-edge-connected graph. We orient some pairwise edge-disjoint cycles of G to directed cycles (by choosing a direction for each). Is it possible to orient the remaining edges to obtain a k-edge-connected orientation of G?

2. Let $D=(V,A)$ be a digraph in which the size of the smallest dicut is k. Prove that the maximal number of disjoint k-sized dicuts equals to the minimal number of edges that covers all the k-sized dicuts.

3. Let $b:2^V\to\mathbb{R}$, where $b(X)+b(Y)\geq b(X\cup Y)+b(X\cap Y)$ whenever $X\cap Y\neq\varnothing$ (i.e., b is intersecting submodular). Show that the function $b':2^V\to\mathbb{R}$ where $b'(X):=\min\{\sum_{Y\in\mathcal{X}}b(Y):\mathcal{X}\text{ is a partition of }X\}$ is submodular and whenever $m\leq b$ is a modular function we have $m\leq b'\leq b$.

4. Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be matroids on the ground set E with rank functions r_1 and r_2 respectively. Show that the following system is TDI and it is the polytope of the characteristic functions of the common independent sets of $\mathcal{M}_1$ and $\mathcal{M}_2$. Derive the Matroid Intersection Theorem from this.

$$x\in\mathbb{R}^S_+$$
$$\sum_{e\in F}x(e)\leq r_i(Z)\quad(i\in\{1,2\},F\subseteq E)$$