1. Suppose that $Ax = \underline{0}$ where $A \in \mathbb{R}^{m\times n},\ x \in \mathbb{R}^n \setminus \{\underline{0}\}$. Show that x is linearly independent from the rows of A.

2. Is it true that $Ax = \underline{0}$ has a nowhere 0 solution if and only if every column of A is a linear combination of the other columns?

3. Pick maximal number of independent rows and maximal number of independent columns in the matrix A. Prove that the corresponding submatrix is quadratic and regular.

4. Let $S \subseteq \mathbb{R}^n$ be a subspace, show that for an appropriate matrix A we have $S = \{x \in \mathbb{R}^n : Ax = \underline{0}\}$.

5. A linear combination $\sum_{i=1}^m \lambda_i v_i$ is **affine combination** if $\sum_{i=1}^m \lambda_i = 1$. If $F \subseteq \mathbb{R}^n$ is nonempty and closed under affine combination, then it is called an **affine subspace**. Let $F \subseteq \mathbb{R}^n$ be an affine subspace.

- Show that there is a linear subspace $S \subseteq \mathbb{R}^n$ and $q \in \mathbb{R}^n$ such that $F = S + q$ (i.e. $\{s + q : s \in S\}$).
- Prove that for some matrix A and vector b we have $F = \{x \in \mathbb{R}^n : Ax = b\}$.

6. Let $A \in \mathbb{R}^{m\times n}$ and $b \in \mathbb{R}^m$. Prove the following: $\nexists x \in \mathbb{R}^n :\ Ax = b$ iff $\exists y \in \mathbb{R}^m : yA = \underline{0},\ yb \neq 0$ (here y is considered as a row-vector).

7. Show that if P is a nonempty polyhedron and for every $x \in P$ and $\lambda > 0$ we have $\lambda x \in P$, then P is a polyhedral cone.

8.* Let P be a quadratic matrix with non-negative components. Assume that in every column the sum of the components is 1. Show that there is an $x \geq \underline{0}$, such that $Px = x$ and $\sum_{i=1}^n x_i = 1$.