Infinite graph theory II: exercises on 19/05/2022

1. Let G be a locally finite connected infinite graph. Show that there is an enumeration $V(G) = \{v_n : n \in \mathbb{N}\}$ such that for every $n \in \mathbb{N}$ there is some $m > n$ with $v_n v_m \in E(G)$. (10p)

2. Show that in an infinite connected graph G there is a 2-way infinite walk using each edge exactly once if and only if the following hold:

   (a) G is countable;

   (b) $d(v)$ is either even or infinite for every $v \in V(G)$;

   (c) There is no even cut with both sides infinite. (10p)