# Infinite graph theory II: exercises on 12/05/2022

1. Show that for every locally finite $G$ the cycle space $\mathcal{C}$ is a closed subspace of the edge space $\mathcal{E} = \mathbb{F}_2^E$ where $\mathbb{F}_2$ carries the discrete topology and $\mathcal{E}$ the product topology.

2. Prove that the characteristic vectors of cuts together with the null vector is a subspace of $\mathcal{E}$.

3. Show that the cut space $\mathcal{B}$ can be generated via thin sums by the cuts of the form $\delta(v)$.

4. Let $k \geq 2$ be fixed, $I = \{0, \dots, 2k-1\}$, $I_e = \{i \in I : i \text{ is even }\}$, $I_o = I \setminus I_e$. Denote by $I^*$ the set of finite sequences from $I$. Let the vertex set $V$ of the digraph is the union of the disjoint sets $\{s_\mu : \mu \in I^*\}$ and $\{t_\mu : \mu \in I^*\}$. The edge set $E$ of the digraph consists of the following edges. For all $\mu \in I^*$ there are $k$ edges in both directions between the two elements of the following pairs: $\{s_\mu, t_{\mu 1}\}$, $\{s_{\mu i}, t_{\mu(i+2)}\}$ $(i = 0, \dots, 2k-3)$, $\{s_{\mu(2k-2)}, t_\mu\}$. Simple directed edges are $(s_\mu, t_{\mu 0}), (t_{\mu i}, s_{\mu(i+1)})_{i \in I_e}, (s_{\mu i}, t_{\mu(i+1)})_{i \in I_o \setminus \{2k-1\}}, (s_{\mu(2k-1)}, t_\mu)$ for all $\mu \in I^*$. Finally $D := (V, E)$ (see figure 1). Prove that $D$ is $k$-edge-connected and every $st$-path shares some edge with every $ts$-path in $D$. (10 points)

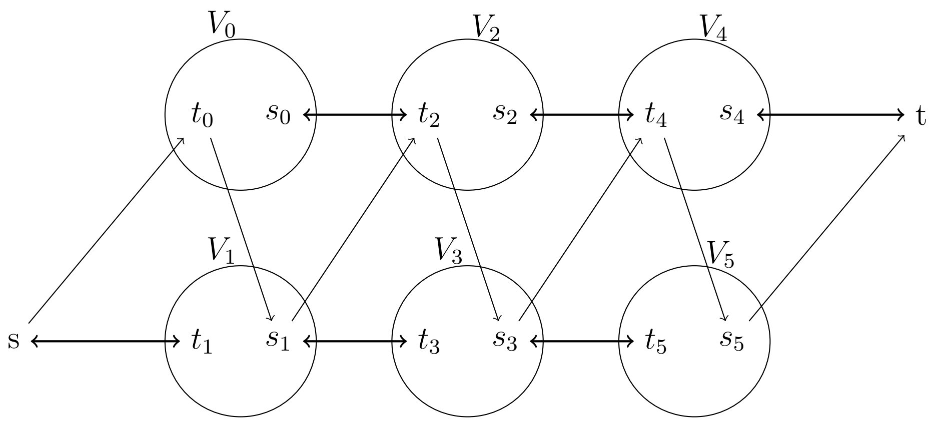


Figure 1: The digraph $D$ in the case $k = 3$. Thick, two-headed arrows stand for $k$ parallel edges in both directions. The (just partially drawn) $D[V_i]$'s are isomorphic to the whole $D$.