Infinite graph theory II: exercises on 21/04/2022

1. Show that if graph G is connected and each $v \in V(G)$ has countable degree, then $V(G)$ is countable.

2. Assume that G is a locally finite graph, $U \subseteq V(G)$ and $\varepsilon \in \Omega(G)$. Prove that G has a comb with its spine in ε and teeth in U if and only if $\varepsilon \in \overline{U}$.

3. Let G be a locally finite graph that admits a normal spanning tree T rooted at r. Prove that every topological end of G has a unique ray in T that starts in r.

4. Does every locally finite 2-connected infinite graph contain an infinite circuit? How about infinite bonds?

5. Show that in every locally finite graph every infinite circuit meets some infinite bond in a single edge.