Game Theory, exercise sheet 7

1. (4 points) Consider the following game:

	A	B	C
A	$(0,0)$	$(6,2)$	$(-1,-1)$
B	$(2,6)$	$(0,0)$	$(3,9)$
C	$(-1,-1)$	$(9,3)$	$(0,0)$

Find two mixed Nash equilibria, one supported on $\{A,B\}$, the other supported on $\{B,C\}$. Show that they are both ESS, but the $\{A,B\}$ equilibrium is not stable when invaded by an arbitrarily small population composed of half B's and half C's.

2. (4 points) Take a two-person general-sum game, and use strong iterated elimination of strategies. If at the end of the process, only one outcome remains, is it true that this outcome is always Pareto-optimal?

3. (4 points) Argue that in a symmetric game, if $a_{ii} > b_{ij}$ $(= a_{ji})$ for all $j \neq i$, then pure strategy i is an evolutionarily stable strategy.

4. (3+3 points) Find all Nash equilibria and determine which of the symmetric equilibria are evolutionarily stable in the following games:

	A	B
A	$(4,4)$	$(2,5)$
B	$(5,2)$	$(3,3)$

	A	B
A	$(4,4)$	$(3,2)$
B	$(2,3)$	$(5,5)$

5. (3+3 points) Occasionally, two parties resolve a dispute (pick a winner) by playing a variant of Rock-Paper-Scissors. In this version, the parties are penalized if there is a delay before a winner is declared; a delay occurs when both players choose the same strategy. The resulting payoff matrix is the following:

	Rock	Paper	Scissors
Rock	$(-1,-1)$	$(0,1)$	$(1,0)$
Paper	$(1,0)$	$(-1,-1)$	$(0,1)$
Scissors	$(0,1)$	$(1,0)$	$(-1,-1)$

a) Show that this game has a unique Nash equilibrium that is fully mixed, and results in expected payoffs of 0 to both players.
b) Show that the following probability distribution is a correlated equilibrium in which the players obtain expected payoffs of 1/2:

	Rock	Paper	Scissors
Rock	0	1/6	1/6
Paper	1/6	0	1/6
Scissors	1/6	1/6	0

6. (3+3 points) Definition: $n+1$ points $v_0, v_1, \ldots, v_n \in \mathbb{R}^n$ are affinely independent if the n vectors $v_i - v_0$, for $1 \leq i \leq n$, are linearly independent.

a) Show that $n+1$ points $v_0, v_1, \ldots, v_n \mathbb{R}^d$ are affinely independent if and only if for every non-zero vector $(\alpha_0, \ldots, \alpha_n)$ for which $\sum_{0 \leq i \leq n} \alpha_i = 0$, it must be that $\sum_{0 \leq i \leq n} \alpha_i v_i \neq 0$.
b) Show that a k-face of an n-simplex is a k-simplex.