Game Theory, exercise sheet 6

Write down and submit the solution for (at least) one of the problems.

1. (3+3 points) A group of 3 students have to do some teamwork and present a project at the end of the semester. They can all be diligent or lazy. If they are all lazy, they fail the semester (utility -5 for everyone), if at least one person does the work properly, they all get a good grade (utility 10 for everyone), but doing the work is tiring, 6 units of work in total, so if they all do their part of the job, that is only -2 utility for them, if two people do it, -3 for them, if one person has to do all the work, that is -6 for him/her.
a) Write down the payoff "matrix". What are the pure Nash equilibria?
b) What are the mixed Nash equilibria?

2. (4 points) Anna, Barbara and Cecilia go on a holiday together. They are arguing about visiting an island, or staying in the city. This is how they are thinking:
Anna: I really want to visit the island. But I won't leave Barbara's side, she does not speak English well and needs my help.
Barbara: I definitely do not want to visit the island, I am afraid of flying. You can go and leave me alone, I don't care.
Cecilia: I prefer the island a bit. I definitely don't want to be alone.

If Cecilia is on the island, the matrix for Anna and Barbara:

		Barbara island	city
Anna	island	$(10, 0, 10)$	$(0, 10, 10)$
	city	$(0, 0, 10)$	$(5, 10, 0)$

If Cecilia is in the city, the matrix for Anna and Barbara:

		Barbara island	city
Anna	island	$(10, 0, 0)$	$(0, 10, 8)$
	city	$(0, 0, 8)$	$(5, 10, 8)$

What are the (pure or mixed) Nash equilibria?

3. (4 points)
Definition: A $\mathbf{s} = (s_1, \ldots, s_k)$ vector of pure strategies is *Pareto-otimal* if there is no other $\mathbf{s}' \in S$ such that $u_i(\mathbf{s}') \geq u_i(\mathbf{s})$ for every player i and $u_j(\mathbf{s}') > u_j(\mathbf{s})$ for at least one player.

Use strong iterated elimination of strategies for the following game. Is the outcome we get Pareto-optimal?

(2,3)	(0,2)	(1,1)
(1,1)	(5,0)	(0,4)

4. (3 points) Consider a k-player game where $\mathbf{x}_i$ is the mixed strategy of player i. For each i, let $T_i = \{s \in S_i | \mathbf{x}_i(s) > 0\}$. Then $(\mathbf{x}_1, \ldots, \mathbf{x}_k)$ is a Nash equilibrium if and only if for each i, there is a constant c_i such that
$\forall s_i \in T_i \; u_i(s_i, \mathbf{x}_{-i}) = c_i$ and $\forall s_i \notin T_i \; u_i(s_i, \mathbf{x}_{-i}) \leq c_i$

5. (4 points) Modify the Hawk and Dove game, now the first player has a third strategy as well: Eagle.

	Hawk	Dove
Hawk	$(0, 0)$	$(4, 1)$
Dove	$(1, 4)$	$(3, 3)$
Eagle	$(-1, 1)$	$(6, -1)$

What are the (pure or mixed) Nash equilibria?

6. (4 points) Show that we can create a linear programming model to decide: for a given strategy, is there a mixed strategy that weakly that dominates it?

7. (4 points) Find all pure or mixed Nash equilibria in the following game:

	X	Y	Z
A	$(3, 4)$	$(5, 3)$	(2,3)
B	$(2, 5)$	$(3, 9)$	(4,6)
C	$(3, 1)$	$(2, 5)$	(7,4)

8. (6 points) We use the following payoffs in prisoner's dilemma, and repeat the game. After each round, it continues to the next round with probability β.

		Prisoner II	
		cooperate	defect
Prisoner I	cooperate	$(6, 6)$	$(0, 8)$
	defect	$(8, 0)$	$(2, 2)$

The Grim strategy in Iterated Prisoner's Dilemma is the following: Cooperate until a round in which the other player defects, and then defect from that point on.

Determine for which values of β it is a Nash equilibrium in Iterated Prisoner's Dilemma for both players to use the Grim strategy.