Game Theory, exercise sheet 5

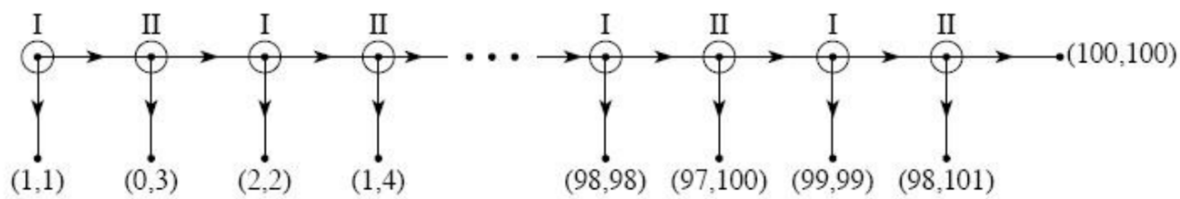
Write down and submit the solution for (at least) one of the problems.

1. (4 points) Two drivers speed head-on toward each other and a collision is bound to occur unless one of them chickens out and swerves at the last minute. If both swerve, everything is ok (in this case, they both get a payoff of 1). If one chickens out and swerves, but the other does not, then it is a great success for the player with iron nerves (yielding a payoff of 2) and a great disgrace for the chicken (a penalty of 1). If both players have iron nerves, disaster strikes (and both incur a large penalty M).

	Swerve	Drive
Swerve	$(1,1)$	$(-1,2)$
Drive	$(2,-1)$	$(-M,-M)$

What are the (pure or mixed) Nash equilibria?

2. (4 points) In the Centripede game, the two players take turns and they decide to leave the path, or go forward. For example, if the first player leaves, they both get 1. If the first player goes forward then the second player leaves, then the first player gets 0, the second player gets 3. The payoffs can be seen on the picture.
This game can be decribed as a strategic game, both players have a finite set of strategies. What is a pure Nash equilibrium here?



3. (4 points) Even for general-sum games, in the strong version of iterated eliminations (eliminating dominated strategies) we do not remove any strategy that appears with a positive probability in a mixed Nash equilibrium.
Is this true? If yes, prove it, if not, give a counterexample.

4. (4 points) The battle of the sexes: A boy and a girl would like to go to a concert. The boy prefers Rammstein, the girl likes Die Prinzen. But they are only happy if they can be together.

		Girl	
		Rammstein	Die Prinzen
Boy	Rammstein	$(3,2)$	$(0,0)$
	Die Prinzen	$(0,0)$	$(2,3)$

What are the (pure or mixed) Nash equilibria?

5. (4 points) Two players play a zero-sum game with the following payoff matrix. Find a Nash equilibrium.

$$\begin{array}{ccc} 0 & 3 & 2 \\ 4 & -1 & 0 \\ 1 & -3 & 1 \end{array}$$

6. (3+3 points)

a) Consider these linear programming problems:

Primal problem		Dual problem	
Maximize	$\mathbf{cx}$	Minimize	$\mathbf{yb}$
subject to	$A\mathbf{x} \le \mathbf{b}$	subject to	$\mathbf{y}A \ge \mathbf{c}$
and	$\mathbf{x} \ge \mathbf{0}$	and	$\mathbf{y} \ge \mathbf{0}$

(The easier direction of the duality theorem:) Supposing that the primal problem has a non-empty set of solutions and $\mathbf{cx}$ is bounded from above, and the dual problem has a non-empty set of solutions and $\mathbf{yb}$ is bounded from below, show that $\max \mathbf{cx} \le \min \mathbf{yb}$.

b)

Primal		Dual	
Maximize	$\mathbf{c_1x_1} + \mathbf{c_2x_2}$	Minimize	$\mathbf{y_1b_1} + \mathbf{y_2b_2}$
subject to	$A\mathbf{x_1} + B\mathbf{x_2} \le \mathbf{b_1}$	subject to	$\mathbf{y_1}A + \mathbf{y_2}C \ge \mathbf{c_1}$
and	$C\mathbf{x_1} + D\mathbf{x_2} = \mathbf{b_2}$	and	$\mathbf{y_1}B + \mathbf{y_2}D = \mathbf{c_2}$
and	$\mathbf{x_1} \ge \mathbf{0}$	and	$\mathbf{y_1} \ge \mathbf{0}$

Supposing that the primal problem has a non-empty set of solutions and $\mathbf{c_1x_1} + \mathbf{c_2x_2}$ is bounded from above, and the dual problem has a non-empty set of solutions and $\mathbf{y_1b_1} + \mathbf{y_2b_2}$ is bounded from below,
show that $\max \mathbf{c_1x_1} + \mathbf{c_2x_2} \le \min \mathbf{y_1b_1} + \mathbf{y_2b_2}$.