Game Theory, exercise sheet 4

Write down and submit the solution for (at least) one of the problems.

1. (3+3 points) **a)** We do the strong version of iterated elimination of dominated strategies. Show that it does not matter in what order we remove the strategies, the set of strategies we have left in the end is always the same.
b) We do the weak version of iterated elimination. Give an example that the set of strategies we have left in the end can be different (depending on elimination order) and the "table" we have left can also be different.

2. (3 points) In the strong version of iterated eliminations we do not remove any strategy that appears in a pure Nash equilibrium.

3. (3+2 points) Two players play Rock Paper Scissors. The payoff matrix is the following.

	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

a) What are the optimal safety strategies for the players?
b) What is a mixed Nash equibrium in this game?

4. (3 points) Two players play a game, player I writes a number on a piece of paper, then tells player II what she wrote, but she can also lie. The player II guesses whether Player I lied of told the truth. If Player II finds out that Player I lied, he receives 10 dollars. If Player II correctly thinks that Player I told the truth, Player II gets 1 dollar. If Player II did not guess correctly, he has to play 6 or 7 dollars to Player I.
The payoffs for Player I:

	Guesses truth	Guesses a lie
Tells truth	-1	7
Lies	6	-10

What are the optimal safety strategies for the players? What is the value of the game?

5. (4 points) Two players play a zero-sum game with the following payoff matrix

$$\begin{matrix} 2 & 3 & 1 & 5 \\ 4 & 1 & 6 & 0 \end{matrix}$$

What are the (pure or mixed) Nash equilibria?

6. (4 points) Two players play a zero-sum game with the following payoff matrix

$$\begin{matrix} 0 & 2 \\ t & 1 \end{matrix}$$

Where t is a parameter (a real number). What are the (pure or mixed) Nash equilibria?

7. (4 points) Two players play a zero-sum game with the following payoff matrix. Find a Nash equilibrium.

$$\begin{matrix} 5 & 4 & 1 & 0 \\ 4 & 3 & 2 & -1 \\ 0 & -1 & 4 & 3 \\ 1 & -2 & 1 & 2 \end{matrix}$$

8. (4 points) On an island, there are k people who have blue eyes, and the rest of the people have green eyes. At the start of the puzzle, no one on the island ever knows their own eye color. By rule, if a person on the island ever discovers they have blue eyes, that person must leave the island at dawn; anyone not making such a discovery always sleeps until after dawn. On the island, each person knows every other person's eye color, there are no reflective surfaces, and there is no communication of eye color.
At some point, an outsider comes to the island, calls together all the people on the island, and makes the following public announcement: "At least one of you has blue eyes". It is common knowledge that the outsider is truthful, and thus it becomes common knowledge that there is at least one islander who has blue eyes. The problem: assuming all persons on the island are completely logical and that this too is common knowledge, what is the eventual outcome?