Game Theory, exercise sheet 3

Write down and submit the solution for (at least) one of the problems.

1. (3 points) Consider the one-pile game with the rule that you may remove at most half of the chips. Of course, you must remove at least one, so the terminal positions are 0 and 1. Find the Sprague-Grundy function.

2. (3 points) Find the Sprague-Grundy function of the (one pile) substraction game with substraction set $S = \{1, 3, 4\}$ (So, in a step a player removes 1 or 3 or 4 chips.)

3. (4 points) We play Hackenbush. Find the Sprague-Grundy values of the graphs in this picture, and find a winning move, if any.



4. (6 points) We play Hackenbush on trees. Create an algorithm to find which edge should we cut next from a given starting position (if there is a winning move).

5. (4 points) We play the Shannon Switching Game where the graph is an $L \times (L+1)$ grid with the vertices of the bottom side merged into a single vertex, A, and the vertices on the top side merged into another node, B. Use a strategy-stealing argument to show that the first player in the Shannon switching game has a winning strategy.



6. (2+2 points) Two players take turns placing dominos on an $n \times 1$ board of squares, where each domino covers two squares and dominos cannot overlap. The last player to play wins.

a) Where would you place the first domino when n = 11?

b) Show that for n even and positive, the first player can guarantee a win.

7. (3 points) At a restaurant, the owner buys one kind of ingredient in the morning. At the same time, the chef decides what he will cook that day (he does not know about the ingredient, and he cannot change his decision later). After the dish is made, the owner pays the chef based on the outcome. (See the payments in the table below). The chef wants to maximalize, the owner wants to minimalize the payment.

Find an equilibrium, which a pair of "pure" choices (one choice for the owner and one choice for the chef) such that both players can think: *if the other player does not strange his strategy*, *I also won't change my strategy*.

Columns: ingredients, the owner chooses

Rows: dishes, the chef chooses

	Garlic	Fish	Chocolate	Lemon
Soup	50	20	10	40
Burger	30	20	50	80
Shake	40	30	70	50
Muffin	20	10	60	10