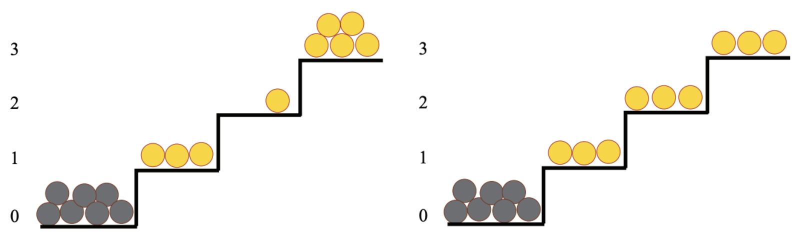
Game Theory, exercise sheet 2

Write down and submit the solution for (at least) one of the problems (you can choose which one).

1. (4 points) There is one heap of n pebbles on the table.
Two players take turns removing pebbles. In the first step, the first player can remove any number of pebbles between 1 and $n-1$. If a player removes k pebbles, the next player may take at least 1, at most $2k-1$ pebbles. The last player who removed something wins.
Who has a winning stategy, the first or the second player, and what is the winning strategy? How does it depend on n?

2. (4 points) Starting with two piles of n and m chips, two players plays the following game: in one step, a player throws away all the chips from one pile, and separates the other one into two non-empty piles. The one who cannot make a move loses (this occurs only when each pile has only one chip).
For which (n, m) pairs does the first player have a winning strategy?

3. (3 points) (Staircase Nim). This game is played on a staircase of n steps. On each step j for $j = 1, \ldots, n$ is a stack of coins of size $x_j \geq 0$. Each player, in his turn, picks j and moves one or more coins from step j to step $j-1$. Coins reaching the ground (step 0) are removed from play. The game ends when all coins are on the ground, and the last player to move wins. Show that the P-positions in Staircase Nim are the positions such that the stacks of coins on the odd-numbered steps have Nim-sum 0.



4. (5 points) A combinatorial game with a position set V is said to be *progressively bounded* if, for every starting position $x \in V$, there is a finite bound $B(x)$ on the number of moves until the game terminates.
The game is *progressively finite* if, for every starting position $x \in V$, the game will end after finite steps.
Show that if every position has a finite outdegree and the game is progressively finite then it is also progressively bounded.

5. (1+1+3 points) Find all the winning moves in the game of nim
a) with three piles of 12, 19 and 27 chips
b) with four piles of 13, 17, 19 and 23 chips
c) What are the answers to (a) and (b) if the misère version of nim is played? (i.e. the one who removes the last chip, loses)

6. (3 points) Consider the take-away game with the rule that you may remove any *even* number of chips of any pile or you may remove any pile consisting of one chip. (The only terminal position is 0.) Find the Sprague-Grundy function.

7. (4 points) We have three piles of chips. In every step, players have to remove at least one chip, but from the first pile, a player may remove at most 3 chips, from the second pile, at most 5 chips, from the third pile at most 6 chips. Who has a winning strategy if the starting position is (19, 24, 15)?

8. (5 points) We have three piles of chips.
From the first pile: you may remove at least 1, at most 5 chips.
Second pile: play the take-away move described in Problem 6.
Third pile: like in nim, you may remove any number of chips, but at least one. Who has a winning strategy if the starting position is (18, 13, 21)?

9. (4 points) Starting with a pile of n chips, two players, I and II, alternate taking a certain number of chips. Player I can remove 1 or 4 chips. Player II can remove 2 or 3 chips. The last player who removes chips wins the game. What are the terminal positions?
Who has a winning strategy, starting with $n = 7$ chips?