## Game Theory, exercise sheet 10

1. (3 points) There are n men and n women, and their preference lists contains all the members of the other sex. Show that in every stable matching is a complete matching, everyone has a partner.

2. (3 points) There are n men and n women, and their preference lists contains all the members of the other sex. Man  $m_1$  and woman  $w_1$  are unpopular, they are the last on everyone's preference lists. Show that in every stable matching,  $m_1$  and  $w_1$  are married to each other.

**3.** (4 points) Stable matchings, preferences by compatibility.

Suppose we seek stable matchings for n men and n women with preference order determined by a matrix  $A = (a_{i,j})$  where all entries in each row are distinct and all entries in each column are distinct. If in the  $i^{th}$  row of the matrix we have  $a_{i,i_1} > a_{i,i_2} > \cdots > a_{i,i_n}$ 

have  $a_{i,j_1} > a_{i,j_2} > \cdots > a_{i,j_n}$ then the preference order of man *i* is  $j_1 > j_2 > \cdots > j_n$ . Similarly, if in the  $j^{th}$  column we have  $a_{i_1j} > a_{i_2j} > \cdots > a_{i_nj}$ ,

then the preference order of woman j is  $i_1 > i_2 > \cdots > i_n$ .

(Imagine that the number  $a_{ij}$  represents the compatibility of man *i* and woman *j*.)

Show that in this case the stable matching is unique.

4. (4 points) Office allocations: A weaker notion of instability than the one discussed in the lecture requires that no set of graduate students can obtain better offices than they are assigned in  $\pi$  by reallocating among themselves the offices allocated to them in  $\pi$ .

Show that this "weak stability" follows from stability.

Give an example that the converse does not hold (i.e. the weaker version of stability does not imply stability)

5. (4 points) Show the that Four Stockholders game (see the lecture slides) has no solution in the core.

**6.** (3+3+1 points) Consider a the game with characteristic function

S	1	2	3	12	13	23	123
v(S)	0	1	2	3	3	4	5

a) What is the core of this game?

**b**) What is the Shapley-value?

c) Is the Shapley value in the core?

7. (4 points) (Market with one seller and two buyers) Player 1 owns an art object of no intrinsic worth to him. Therefore he wishes to sell it. The object is worth 30 dollars to player 2 and 40 dollars to player 3. Set this up as a game in characteristic function form. Find the Shapley value. Is the Shapley value in the core?

8. (3+3+3 points) Find the Shapley value of the *n*-person game, for n > 2, with characteristic function

a) v(S) = |S| if  $1 \in S$ , and v(S) = 0 otherwise.

**b)** v(S) = |S| if  $1 \in S$  or  $2 \in S$ , and v(S) = 0 otherwise.

c) v(S) = |S| if  $1 \in S$  and  $2 \in S$ , and v(S) = 0 otherwise.