

Game Theory, exercise sheet 1

1. There is one heap of n pebbles on the table.

Two players take turns removing pebbles. In one step a player can remove at least one, at most four pebbles. The last player to take any objects is the winner.

Who has a winning strategy, the first or the second player, and what is the winning strategy? How does it depend on n ?

2. There are two heaps of pebbles on the table. There are n and m pebbles in them.

Two players, take turns removing pebbles from the two piles. In one step a player can remove any number of pebbles (at least one) from the same pile.

The last player to take any pebbles is the winner.

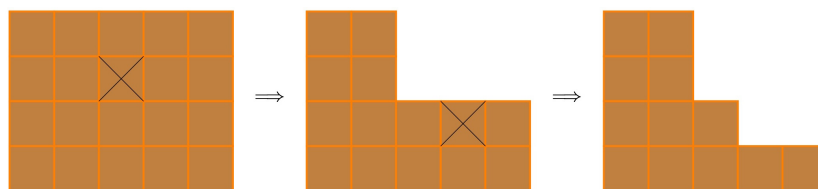
Who has a winning strategy, the first or the second player, and what is the winning strategy? How does it depend on n and m ?

3. The same game (and same question) as in Problem 2 but the person who removes the last pebble *loses*.

4. In the game of Chomp, two players start with a chocolate bar, which is scored into an a by b array of squares (where $a \cdot b > 1$). The square in the lower left is poisoned.

The players alternate turns. On their turn, a player chooses a square, then eats it, along with all of the squares which are either above it, to its right, or both. This continues until someone eats the poisoned square, and the non-poisoned player wins.

Here is an example of how the first two turns might go, where the "x" indicates the square that player chose:



Find who has the winning strategy and what is the winning strategy:

- a) for a $2 \times n$ chocolate table.
- b) for an $n \times n$ chocolate table.

5. Two players play the following game: they agree on a positive integer N .

On each player's turn, that player writes down a divisor of N , but they are not allowed write a divisor of any number that was already written down. The player who writes N loses.

(For example: they agree on 20 and they write 2, 4, 10, 20. In this example, the second player lost.)

- a) Find the winning strategy for $N = 20$.
- b) Find the winning strategy for $N = 72$.
- c) In general, who has a winning strategy? How does it depend on N ?