Discrete Mathematics, exercise sheet 8

- **1.** (2 points) Is the complement of the cycle of length 6 (C_6) a planar graph?
- 2. (3 points) Show that the Petersen graph is not planar.



3. (2 points each) a) Show that the edges of the Petersen graph cannot be colored with 3 colors.
b) Show that the Petersen graph does not have a Hamiltonian cycle, but deleting any vertex, the remaining graph has a Hamiltonian cycle.

4. (4 points) A group of musicians are traveling. Everyone has 3 enemies in the group. Show that they can be divided to sit on two buses in a way that everyone has at most one enemy who is traveling on the same bus as him.

5. (4 points) A *regular polyhedron* is a (3 dimensional) polyhedron whose faces are identical regular polygons. All side lengths are equal, and all angles are equal. In every vertex the same number of faces meet. Using Euler's Formula, show that only five convex regular polyhedra exist.

6. (2 points) Is there a bipartite graph with degrees 3,3,3,3,3,3,3,3,3,3,5,6,6? (These can be distributed in the two classes of nodes arbitrarily.)

7. (2 points) An island is inhabited by six tribes. They are on good terms and split up the island between them, so that each tribe has a hunting territory of 100 square miles. The whole island has an area of 600 miles. The tribes decide that they all should choose new totems. They decide that each tribe should pick one of the six species of tortoise that live on the island. They want to pick different totems, and totem for each tribe should occur somewhere on their territory. The areas where the different species of tortoises live don't overlap, and they have they same area - 100 square miles. (Of course, the way the tortoises divide up the islands may be entirely different from the way way the tribes do.) Show that such a selection of totems is always possible.

8. For handing in. (10 points) Determine the number of pieces into which a circle is divided if n points on its circumference are joined by all possible chords. The chords are in a general position, no three of them goes thought the same point.