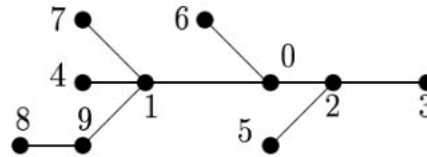
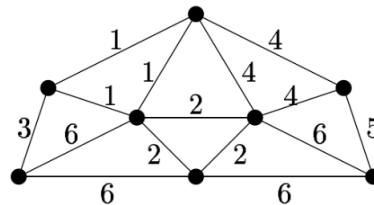


Discrete Mathematics, exercise sheet 7

1. (2 points) Give the Prüfer code of the following tree:



2. (2 points) Select the value of x such that $1, 1, 5, x, 6, 6$ is a Prüfer code of a tree, in which every degree is odd. Give the tree as well.
3. (2 points) Show that if a tree has a k -degree node, then it has at least k leaves. Is the reverse statement true?
4. (2 points) How many trees are there on n labelled nodes, that have at least 3 leaves?
5. (2 points) Find a minimum cost spanning tree of this graph. How many minimum cost spanning trees are there?



6. (2 points) G is a simple graph, its vertices are labelled with $1, 2, \dots, 100$. Nodes i and j are connected by an edge in G if and only if $|i - j| \leq 2$. Does G contain an Eulerian circuit or an Eulerian walk?
7. (2 points) Is there a graph on 10 nodes that contains an Eulerian circuit and the sum of the degrees is 34?
8. (1+1 point) **a)** Find a graph, where every degree is even, and it does not contain an Eulerian circuit.
b) Find a graph that is not connected, and contains an Eulerian circuit.
9. (4 points) In a group everyone knows 4 other people. (We assume that acquaintance is mutual.) Show that they can sit down around some round tables in a way that everyone knows his/her two neighbors.
10. (5 points) A government wants to connect cities with roads, (i. e. they want to build a spanning tree). Optimists and pessimists win in unpredictable order. This means that sometimes they build the cheapest line that does not create a cycle with those lines already constructed; sometimes they mark the most expensive lines “impossible” until they get to a line that cannot be marked impossible without disconnecting the network, and then they build it. Prove that they still end up with an optimal cost spanning tree.
11. **For handing in.** (6 points) Tree T has 17 nodes and the degree of each node is either 1 or 4. After Alice added some edges to this graph, it has an Eulerian circuit. At least how many edges did she add?