Discrete Mathematics, exercise sheet 6

1. (2 points) In a group of 8 people, some of them shake hands. Is it possible that everyone shaked hands with a different number of people?

2. (2 points) In a simple, connected graph on 6 vertices, the degrees of 5 vertices are 1, 2, 3, 4, 5 respectively. What may be the degree of the 6^{th} vertex?

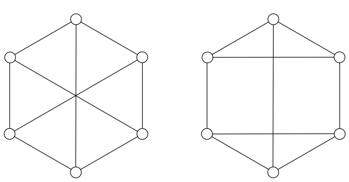
3. (2 points) Draw all simple graphs on 4, 5, or 6 vertixes that are isomorphic to their complement.
(The *complement* of a graph G is a graph $\overline{G}$ on the same vertices such that two distinct vertices of $\overline{G}$ are connected by an edge if and only if they are not connected by an edge in G.
Two graphs are *isomorphic* if there exists a one-to-one correpondence between the nodes of the first graph and the nodes of the second graph so that two nodes in the first graph that are connected by an edge correspond to nodes in the second graph that are connected by an edge, and vice versa.)

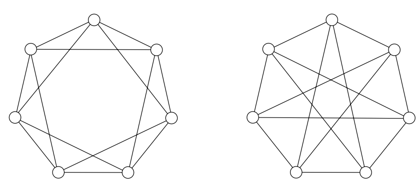
4. (1 point each) Is there a simple graph where the degrees of the vertices are
a) 3, 3, 3, 2, 2, 2, 1, 1, 1; **b)** 6, 6, 5, 4, 4, 3, 2, 2, 1;
c) 7, 7, 7, 6, 6, 6, 5, 5, 5; **d)** 1, 3, 3, 4, 5, 6, 6?

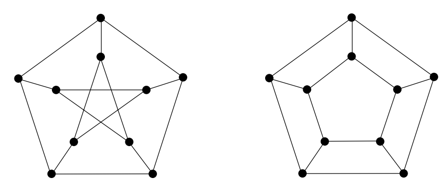
5. (2 points) In a simple graph, vertex v has an odd degree. Prove there is a path from v to another vertex with odd degree.

6. (3 points) Characterize the graphs with the following property: any two edges have a common endpoint.

7. (3 points) Which pairs of graphs are isomorphic?

a) 

b) 

c) 

8. (2 points) Draw the tree with the Prüfer code 4 3 0 1 1 3.

9. (5 points) At most how many intersections do the diagonals of a convex n-sided polygon have?

10. For handing in. How many trees are there on n labeled vertices such that
a) (3 points) the degree of each node is at most 2.
b) (5 points) the node with label 1 has degree 1.