Discrete Mathematics, exercise sheet 4

For problems 1-2, each subproblem is worth 1 point, except for 1/e.

1. In a German lottery, players are required to choose six main numbers between 1 and 49 plus an additional number, known as the Superzahl, between 0 and 9. To win the jackpot, a player must match all seven numbers, but prizes are available for matching as few as two main numbers plus the Superzahl.
a) What is the probability of getting all seven numbers right?
b) What is the probability of getting 6 numbers right, but not the Superzahl?
c) What is the probability of getting the 6 numbers right? (We don't care about the Superzahl)
d) What is the probability getting exactly 5 numbers of the main 6 right? (We don't care about the Superzahl)
e) (2 p) What is the probability getting at least 3 numbers of the main 6 right, and getting the Superzahl wrong?

2. There are 10 red, 20 yellow and 40 green balls in a box. With pick some of them with closed eyes, and do not put them back. At least how many balls should we pick, to surely have
a) one yellow ball?
b) three balls with different colors?
c) three balls of the same color?
d) 15 balls of the same color?
e) two green balls that were drawn right after each other?

3. (1 point) Show that

$$\binom{n}{k+1} = \frac{n-k}{k+1}\binom{n}{k}$$

4. (4 points) Prove the following equality

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

5. (2 points) How many ways can we cover a $2 \times n$ "chessboard" with 1×2 dominoes?

6. (3 points) Show that the product of n consecutive positive integers is always divisible by $n!$.

7. (2 points) How many ways can we choose three different numbers from the set $\{1, 2, 3, \ldots 100\}$ in a way that the sum of these three numbers is divisible by 3?

8. For handing in. (7 points)
Prove that for the Fibonacci numbers $F_0 + F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$ for every $n \geq 0$.