## Discrete Mathematics, week 3

For problems 1-4, each subproblem is worth 1 point.

- 1. a) There is a building with *n* floors (counting the ground floor as well).
- How many ways can we paint the levels to red, yellow or blue?
- b) What happens if two consecutive levels cannot have the same color?
- 2. a) How many ways can a lion, a pengiun, a tiger and a polar bear stand in a row?
- **b)** What if we have one more lion?
- c) What if we have yet one more lion?
- d) We have 4 lions, 2 tigers and 3 polar bears. (We do not distinguish between animals of the same species.)
- **3.** On a  $8 \times 8$  chessboard, how many ways can we place
- a) one black and one white stones; b) two white stones;
- $\mathbf{c}$ ) one black, one white, and one green stone;  $\mathbf{d}$ ) three white stones;
- e) three black and four white stones?

4. How many 8-digit numbers are there? How many 8-digit number are there with the following property:

- a) the consecutive digits are different.
- **b)** it does not contain the digit 5.
- c) it contains the digit 5.
- d) there are two digits that are the same (there may be more).
- e) there are two *consecutive* digits that are the same (there may be more).
- f) there are exactly two digits that are the same.
- g) there are exactly two *consecutive* digits that are the same.

**5.** (2 points) Alice goes the florist, and would like to buy 7 flowers. The shop has roses, tulips and carnations. How many ways can she buy 7 flowers? (We do not distinguish between flowers of the same type.)

6. (2 points) George is in Manhattan, and he wants to walk from the corner of  $8^{th}$  Avenue and  $42^{nd}$  Street to the corner of  $11^{th}$  Avenue and  $57^{th}$  Street. He wants to walk one of the shortest possible paths. How many ways can he do it? (The streets and avenues form a grid.)

7. (3 points) How many ways can n boys and n girls stand in a line, if two boys cannot stand next to each other, and two girls cannot stand next to each other?

8. (5 points) 13 green, 15 gray, and 17 red chameleons live in Madagascar. They always meet in pairs. They are easily frightened, if two chameleons of different colors meet, they get so frightened that they both switch to the third color. Is it possible, that after some time, all of them acquire the same color?

## 9. For handing in.

a) (4 points) How many ways can we place 8 rooks on a  $8 \times 8$  chessboard, such that no pair of them can capture each other? (A rook can capture an another rook if they are in the same row or in the same coloumn.) b) (6 points) What if they are not allowed to capture each other and the arrangement of the pieces should be centrally symmetric to the center of the chessboard?