## Discrete Mathematics, exercise sheet 10

1. (3 points) Prove that if we color the points of the Fano plane with 2 colors, there will be line where all three points have the same color.

**2.** (2 points) In a town, there are 924 clubs, and every club has 21 members. Every 2 people can meet each other in exactly 2 clubs. How many inhabitants are in this town? One person is a member of how many clubs?

**3.** (4 points) In a town, the clubs form a block design and every club has a badge. On a big event, everyone from the town is present, and everyone wears a badge of a club he/she is a member of. (Each person wears only one badge.) Is it always possible that everyone wears different badges?

4. (2 points) Show that in a block design with k = 3 and  $\lambda = 1$ , the residue of v divided by 6 is 1 or 3.

5. (3 points) Can you create a block design with the following parameters?  $v = 13, k = 3, \lambda = 1$ .

6. (3 points) We color the points of the  $\mathbb{R}^2$  plane with 3 colors. Show that there are two points such that their distance is 1, and they have the same color.

7. For handing in. (8 points) In a group, everyone has 3 friends. (We assume that friendship is mutual.) If A and B are not friends, there is exactly one person in the group that they are both friends with. If A and B are friends, then they do not have a common friend in the group. Is this situation possible? If it is possible, how many people are in the group?