Discrete Mathematics, week 1

1. Find relations R, S on some set X such that $R \circ S \neq S \circ R$.

2. Let us imagine we intend to buy a refrigerator. We simplify the complicated real situation by a mathematical abstraction, and we suppose that we only look at three numerical parameters of refrigerators: their cost, electricity consumption, and the volume of the inner space. If we consider two types of refrigerators, and if the first type is more expensive, consumes more power, and a smaller amount of food fits into it, then the second type can be considered a better one—a large majority of buyers of refrigerators would agree with that.

The relation "to be clearly worse" (denote it by \leq) in this sense is the following: on the set of triples (c, p, v) of real numbers (c stands for cost, p for power consumption, and v for volume), defined as follows: $(c_1, p_1, v_1) \leq (c_2, p_2, v_2)$ if and only if $c_1 \geq c_2, p_1 \geq p_2$, and $v_1 \leq v_2$.

Show that this \leq is a partial ordering.

3. Are the following relations R over set X equivalence relations?

- a) $X = \mathbb{R}^2$, $((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2$.
- b) $X = \mathbb{R}^2$, $((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1 \cdot y_2 = x_2 \cdot y_1$.

c)
$$X = \mathbb{R}^2 \setminus \{(0,0)\}, ((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1 \cdot y_2 = x_2 \cdot y_1.$$

4. Let R and S be arbitrary equivalences on a set X. Decide which of the following relations are necessarily also equivalences (if yes, prove; if not, give a counterexample).

- a) $R \cap S$
- b) $R \cup S$
- c) $R \setminus S$
- d) $R \circ S$

5. The following (false) proof tries to prove that every symmetric and transitive relation is also reflexive:

Let R be symmetric and transitive relation on set X, then for every $x, y \in X$ with $(x, y) \in R$ since the symmetry $(y, x) \in R$ and using transitivity, $(x, y) \in R$ and $(y, x) \in R$ therefore $(x, x) \in R$. Thus the relation R is an equivalence relation.

Give a counterexample for this statement, and show where is the mistake.