Discrete Mathematics **Comprehensive Exam 2**

17.09.2019. (100 pts, 120 mins)

Using any written material, calculators or mobile phones is not allowed. Please turn off your phone.
Use only paper and pen.
You can use any theorems or statements from the lecture (without proof) if you state them properly. Except if the exercise is to prove that theorem.

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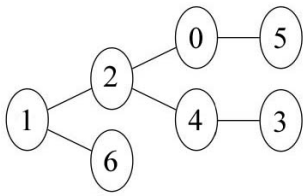
1. (4+6+6 points)
Alice wants to send postcards to 12 friends. In the shop there are only 3 kinds of postcards. In how many ways can she send the postcards, if
a) there is a large number of each kind of postcard, and she wants to send one card to each friend;
b) there is a large number of each kind of postcard, and she is willing to send one or more postcards to each friend (but no one should get two identical cards);
c) the shop has only 4 of each kind of postcard, and she wants to send one card to each friend?

2. (6+8 points)
a) Give the Prüfer code of the following tree.



b) Draw the tree with the Prüfer code 5 2 1 5 0 1 3.

3. (10 points)
Let G be a simple graph, such that each vertex of G has degree 3. Show that if the edges of G cannot have a proper edge-coloring with 3 colors, then G does not have a Hamiltonian cycle.

4. (15 points)
Prove Mirsky's theorem: For any finite partially ordered set P, $\omega(P)$ equals the minimum number of antichains into which the poset may be partitioned.
(Here, $\omega(P)$ denotes the maximum length of a chain in the poset P.)

5. (15 points)
In how many ways can we get exactly 25 points when rolling four 12-sided dice?

6. (15 points)
Let $G = (A, B; E)$ be a bipartite graph with the same number of nodes on both sides. Suppose that every nonempty subset $X \subseteq A$ has at least $|X| - 1$ neighbors. Prove that G contains a matching that matches up all but one node on each side.

7. (15 points)
The inhabitants of a town form clubs. Every club has exactly 3 people, and every two people meet in exactly one club. If the town has v inhabitants, show that the residue of v divided by 6 is 1 or 3.