# **Discrete Mathematics**

16.07.2019. (100 pts, 120 mins)

Using any written material, calculators or mobile phones is not allowed. Please turn off your phone. Use only paper and pen.

You can use any theorems or statements from the lecture (without proof) if you state them properly. Except if the exercise is to prove that theorem.

**1.** (5+5 points)

a) How many five digit numbers are there with 4 even and 1 odd digits?

**b)** How many six digit numbers are there such that all of its digits are different, and it has 4 even and 2 odd digits?

- **2.** (3+8+3 points)
- a) What is the definition of a tree?

b) Prove that if an undirected finite graph has n vertices and n-1 edges and it is connected, then it is a tree.

c) How many trees are there on n labelled vertices? (without proof)

**3.** (4+14 points)

- a) Describe Kruskal's algorithm.
- b) Prove that Kruskal's algorithm always gives a minimum cost spanning tree.
- **4.** (10 points)

State the Max flow - Min cut theorem. (without proof)

## **5.** (14 points)

How many ways are there to seat n married couples at a round table with 2n chairs in such a way that no husband and wife sit next to each other? (Solutions that can be transformed to each other via rotation or reflection still count as different solutions.)

## **6.** (10 points)

At a dance party, 10 boys and 10 girls are present. We want to organize rounds, in each round 10 pairs dance. Any boy and any girl dance with each other only once. Four rounds already happened. Show that we can organize the participants into pairs for the remaining 6 rounds.

## **7.** (12 points)

There are 10 red balls, 10 blue balls, and 10 green balls. In how many different ways can you pick 16 balls such that there is at least one ball for each color?

## 8. (12 points)

The inhabitants of a town like to form clubs. Every club has the same size. Each citizen A must behave "equally" toward citizens B and C, so A must meet B in the same number of clubs as she meets C. Show that this implies that all the people are members of the same number of clubs.