Gödel's incompleteness theorems - The limits of the formal method

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In my talk I am going to give a presentation of the famous incompleteness theorems. After the discovery of several apparent paradoxes in attempted foundations of mathematics (most famously the Russel paradox), around 1922 David Hilbert argued for a method to found mathematics in an undeniably consistent way – today known as Hilbert's programme – which roughly consisted of the following two steps. The first step is to formalize all existing mathematics using suitable axioms and specifying a complete set of valid rules for proofs, from which every mathematical question can be decided. The second step then was to show by purely finitistic means (as they are used e.g. in finite combinatorics) that the formal system specified in this way cannot produce contradictions. In 1933, however, Kurt Gödel proved his famous two incompleteness theorems. The first incompleteness theorem implies that step one of Hilbert's programme cannot be carried out and the second incompleteness theorem implies the same about step two of Hilbert's programme. This puts a hard limit on the possibility to found mathematics in a purely formal way which arguably lays the basis for a large part of modern mathematical logic, in particular proof theory and certain parts of set theory.

I plan to explain the first incompleteness theorem together with a sketch of the most important parts of its proof and to clarify what exactly this theorem means for mathematical practice. If time permits I will also introduce the second incompleteness theorem without going into much detail about how it is proved. The talk should be accessible to any mathematician and no prior experience with mathematical logic is required to understand it.