Cobordism Conjecture - First encounter

following McNamara, Vafa, arXiv:1909.10355

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Cobordism Conjecture in Quantum Gravity

Previous ZMP seminars:

No global symmetry (NGS) conjecture in Quantum Gravity

Cobordism Conjecture:

Powerful reformulation of the NGS conjecture in the language of cobordism theory

Rough idea:

Absence of generalised global symmetries implies - in a sense - uniqueness of QG.

Cobordism group of Quantum Gravity: $\Omega^{\mathrm{QG},D}$

Consider collection of all configurations of a theory of QG with D large spacetime dimensions.

Definition 1:

Two configurations in this collection are called equivalent if it is possible to deform one theory into another by an allowed dynamical process of finite energy

$\Longleftrightarrow$ they are connected by finite energy domain walls.

$\Omega^{\mathrm{QG},D}$ is the set of equivalence classes with respect to this equivalence relation.

Cobordism group Ω_k^{QG}

Compactify d-dimensional QG on k-dimensional background M^k.

$\Longrightarrow$ $\Omega^{\mathrm{QG},D}$ induces a notion of equivalence classes between two such backgrounds.

Definition 2:

Two backgrounds M^k, N^k are cobordant ($M^k \sim N^k$) if we can transition from one to the other by a sequence of dynamically allowed processes in QG.

$$\Omega_k^{\mathrm{QG}} = \{\text{compact, closed k-dim. backgrounds}\}/\sim$$

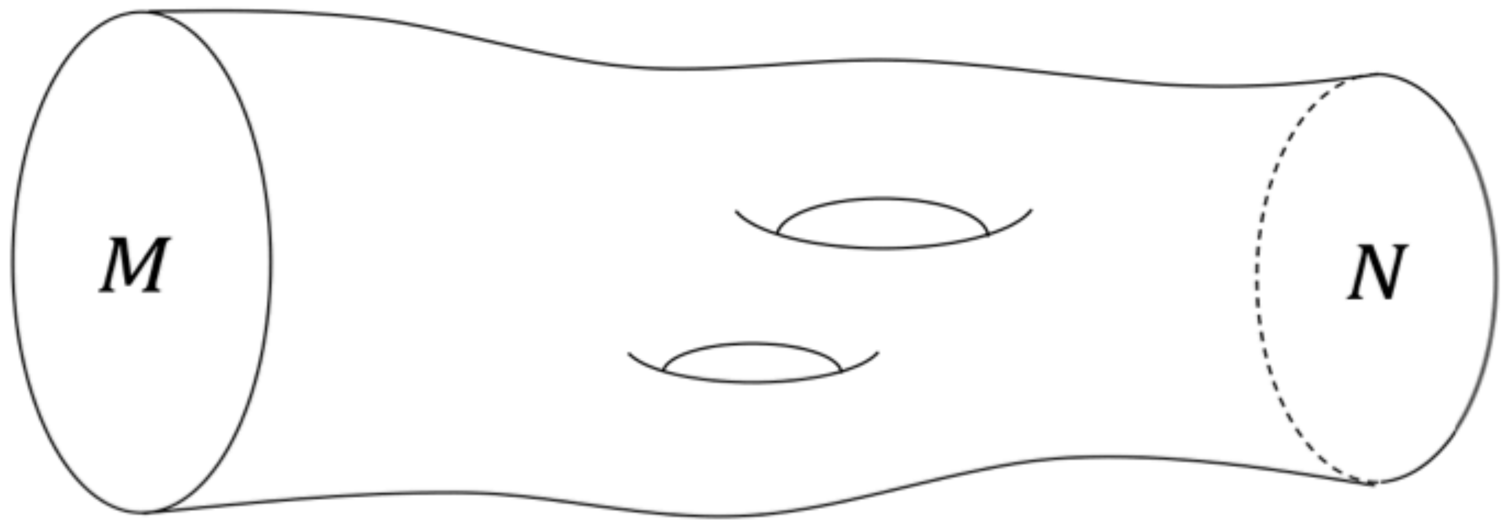
Interpretation as domain walls

Mostly focus (as approximation) on smooth backgrounds with additional structures (such as orientation, spin, ...).

$\Longrightarrow$ concept of **cobordism group** as familiar from mathematics (see later):

$$M^k \sim N^k \text{ iff } M^k \sqcup \bar{N}_k = \partial W^{k+1}$$

domain wall of dimension $(d-k-1)$ in $(d-k)-$dim.



$(d-k)$-dim. $(d-k)$-dim.

$(d-k-1)$-dim.

M^k as $(d-k-1)$ defect

The set of compact, closed k-manifolds gives rise to $(d-k-1)$ defects:

- Consider $\mathbb{R}^k \subset$ d-dim. spacetime.

- Form connected sum $\mathbb{R}^k \# M^k$ by excising a ball and gluing in M^k.

- This glued in region looks like a defect from the perspective of the $D = d-k$ dim remaining dimensions.

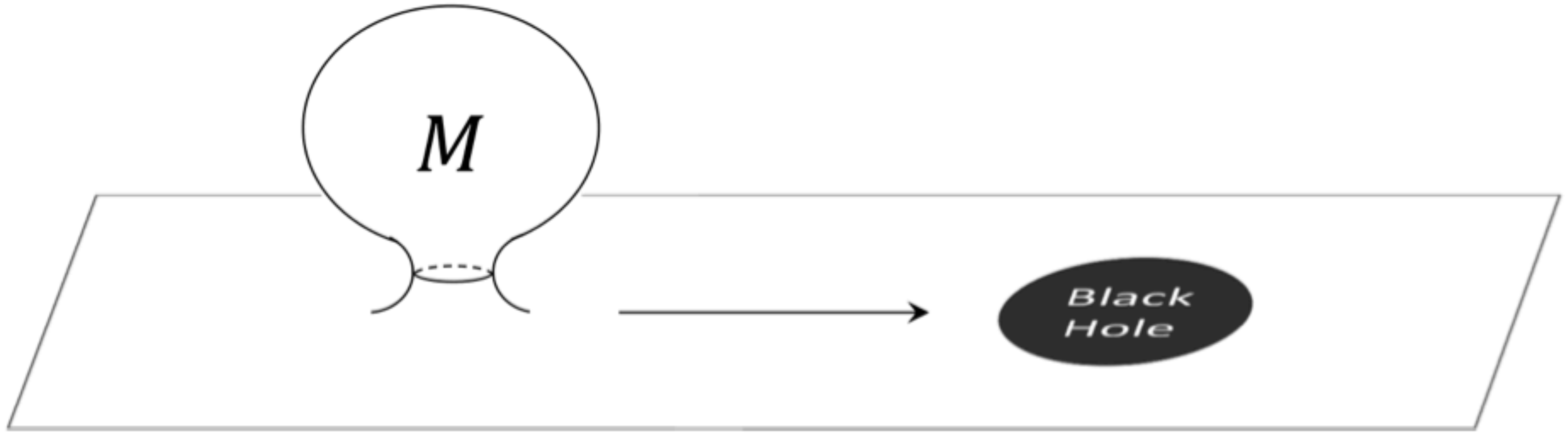
Cobordism Conjecture

Such M^k carries charge under $(d-k-1)$ form symmetry if there exists a property of M^k that does not change under evolution in $(k+1)$ dimensions, i.e. if $[M^k] \neq 0.$

This is a global (not a gauge) charge:

There is no way to detect presence of M^k from outside the region,

i.e. a priori no coupling to a long-range gauge field.



$\Longrightarrow$ forbidden by no-global symmetry conjecture for $(d-k-1)-$ form symmetries

Cobordism Conjecture

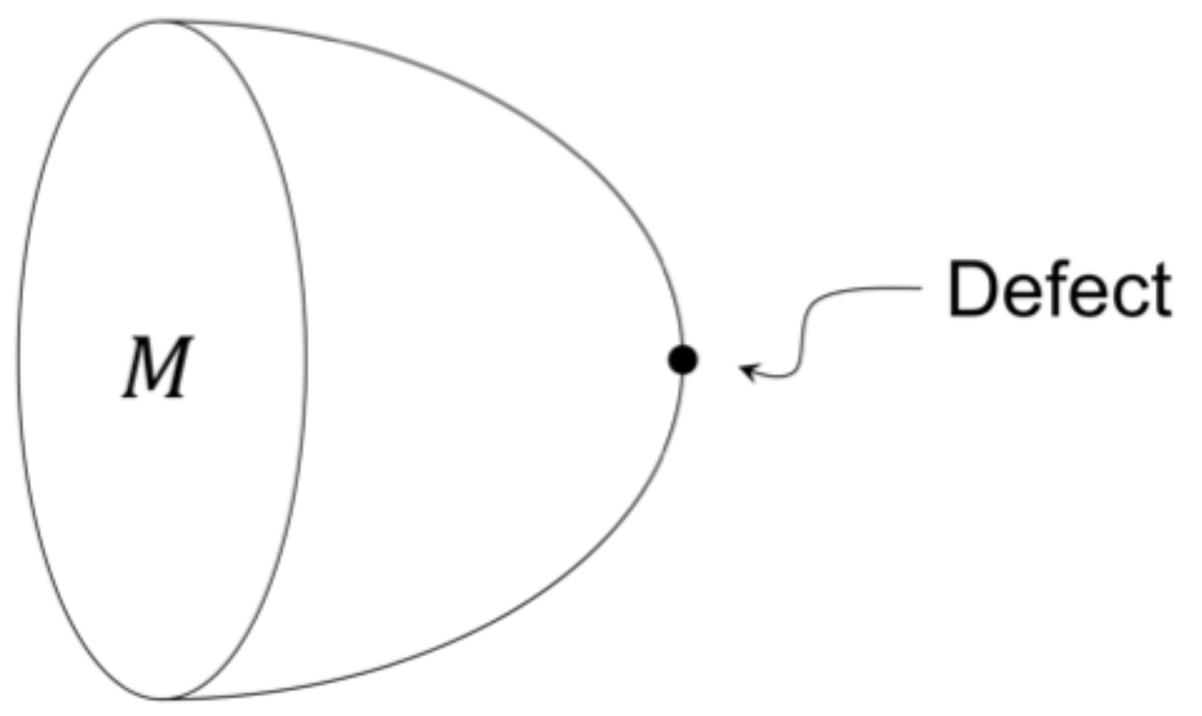
$$\boxed{\Omega_k^{\mathrm{QG}} = \varnothing}$$

McNamara, Vafa, 2019

Take $\widetilde{\mathrm{QG}}$ = approximation to QG with some known structure.
If $\Omega_k^{\widetilde{\mathrm{QG}}} \neq \varnothing$, then modify theory in one of two ways:

i) Add a defect to break the global symmetry:

$$\Omega_k^{\widetilde{\mathrm{QG}}+\mathrm{defect}} = \varnothing$$



ii) Add a $(d-k)$ form gauge field to gauge the symmetry:
Then Gauss's law requires $[M^k] = 0$ (tadpole constraint)