Bordisms in Quantum Gravity

Based on work with:

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- ZMP Colloquium -

Hamburg — June 19, 2025

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CERN

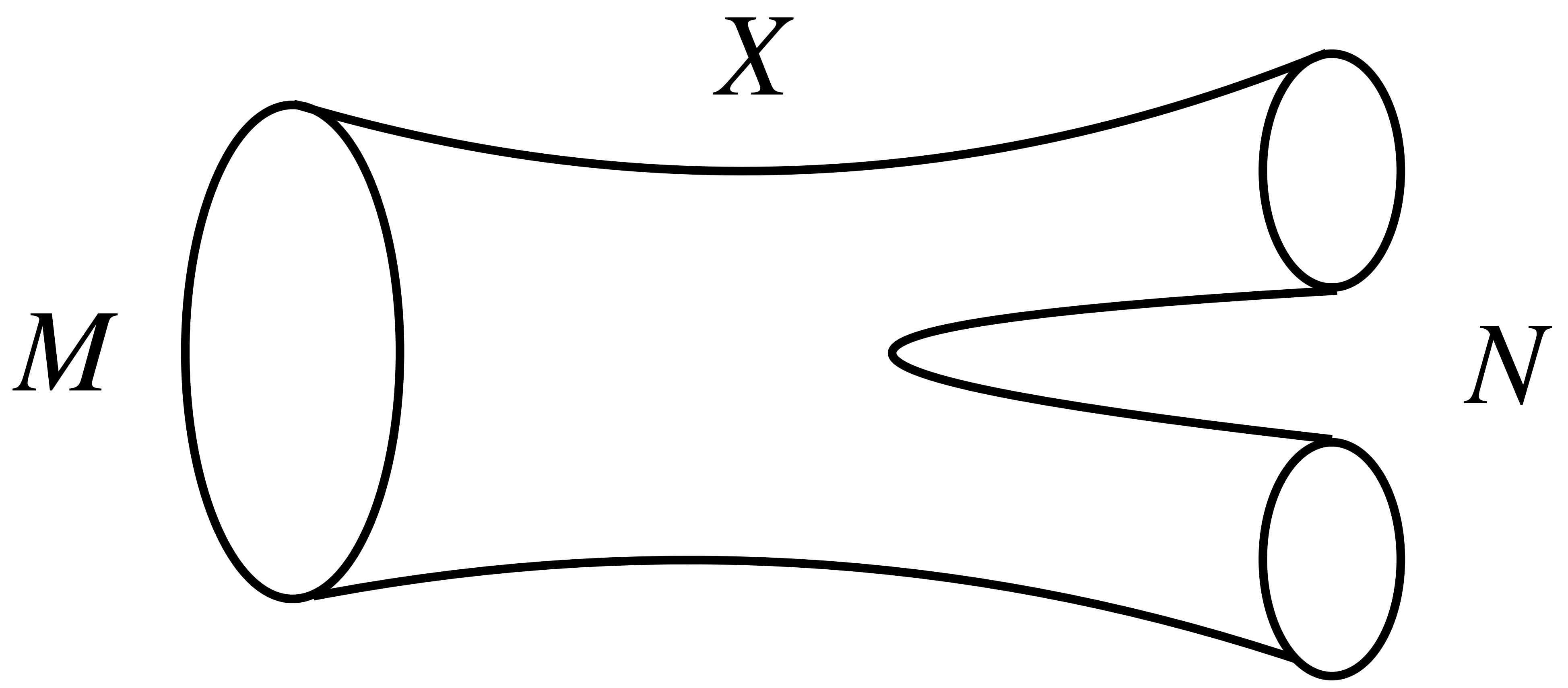
Bordisms

Bordisms (often called cobordisms)

Generalized homology theory (leads to equivalence relation)

Imagine you have two d-dimensional compact manifolds.

Can I find a (d+1)-dimensional one that connects them?



$\partial X = M \sqcup -N$

The deformation classes that cannot be connected are generators of:

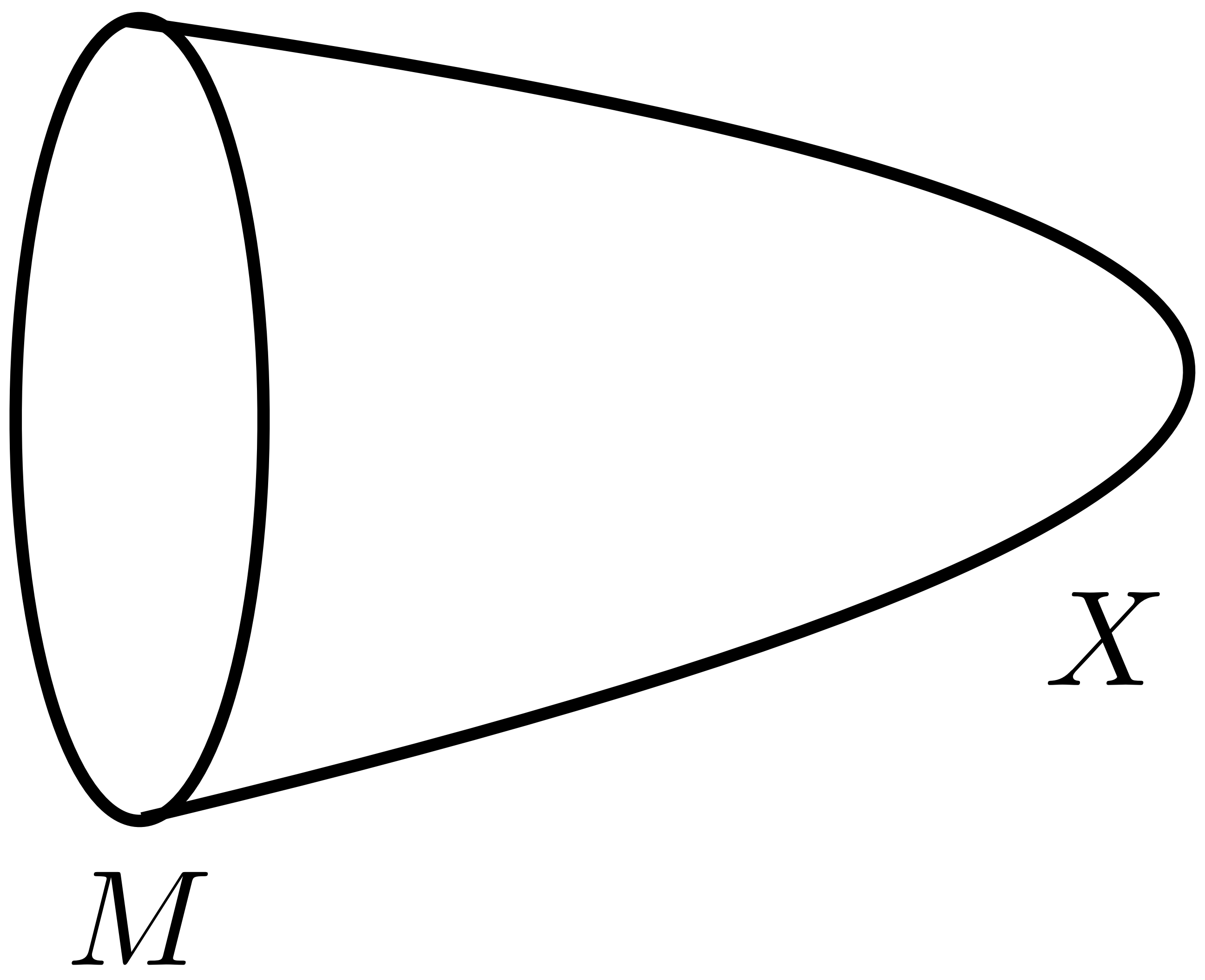
bordism group Ω_d

Triviality

The trivial class is the empty manifold $\emptyset$

$$M \simeq \emptyset$$

if M is a **boundary of a (d+1)-dimensional manifold** $M = \partial X$



A lot of additional data

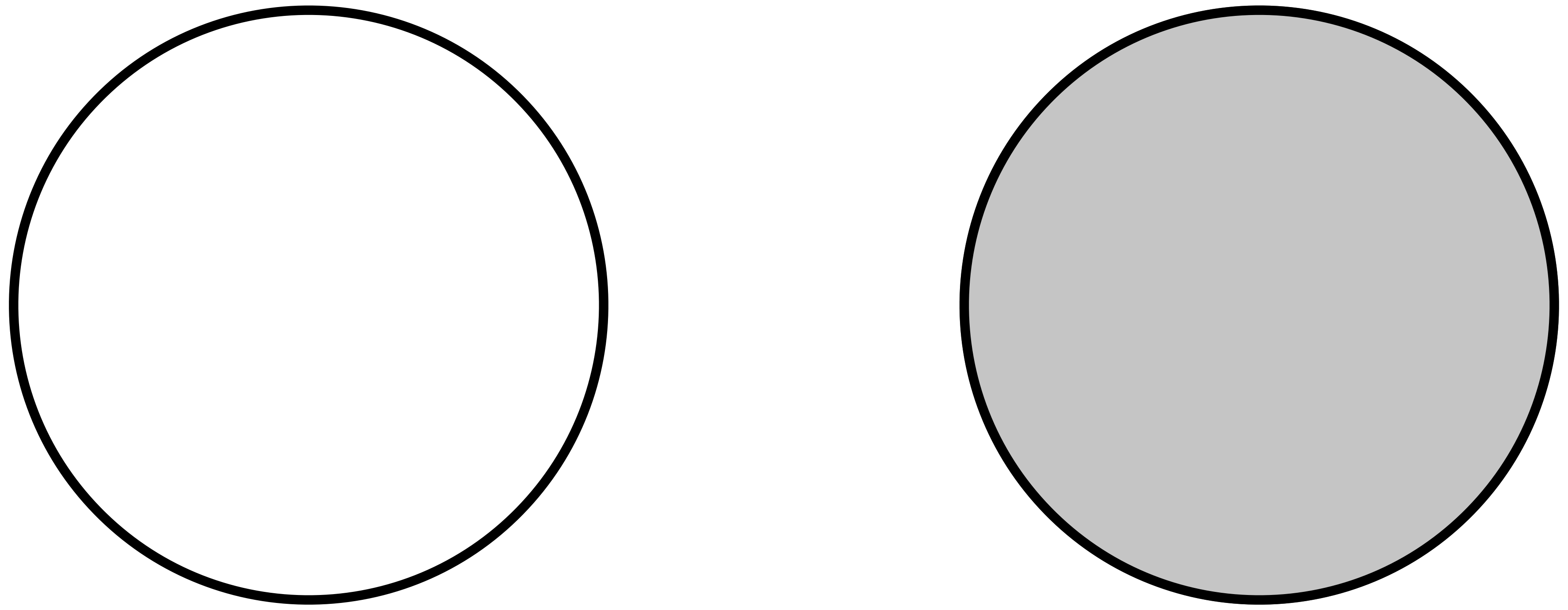
Requirements on manifolds (+ compatibility: bulk and boundary):

- **Smooth** ("low-energy" approximation)
- **Orientation:** $w_1 = 0 \in H^2(X;\mathbb{Z}_2)$ $\Omega_d^{\mathrm{SO}}(pt)$
- **Spin:** $w_2 = 0 \in H^2(X;\mathbb{Z}_2)$ $\Omega_d^{\mathrm{Spin}}(pt)$
- **Additional gauge theory:** $\Omega_d^{\xi}(BG)$
- **Mixing:** $\Omega_d^{\mathrm{Spin}-G}(pt)$

  (e.g. Spin^c requires $w_2 = c_1 \mod 2$)

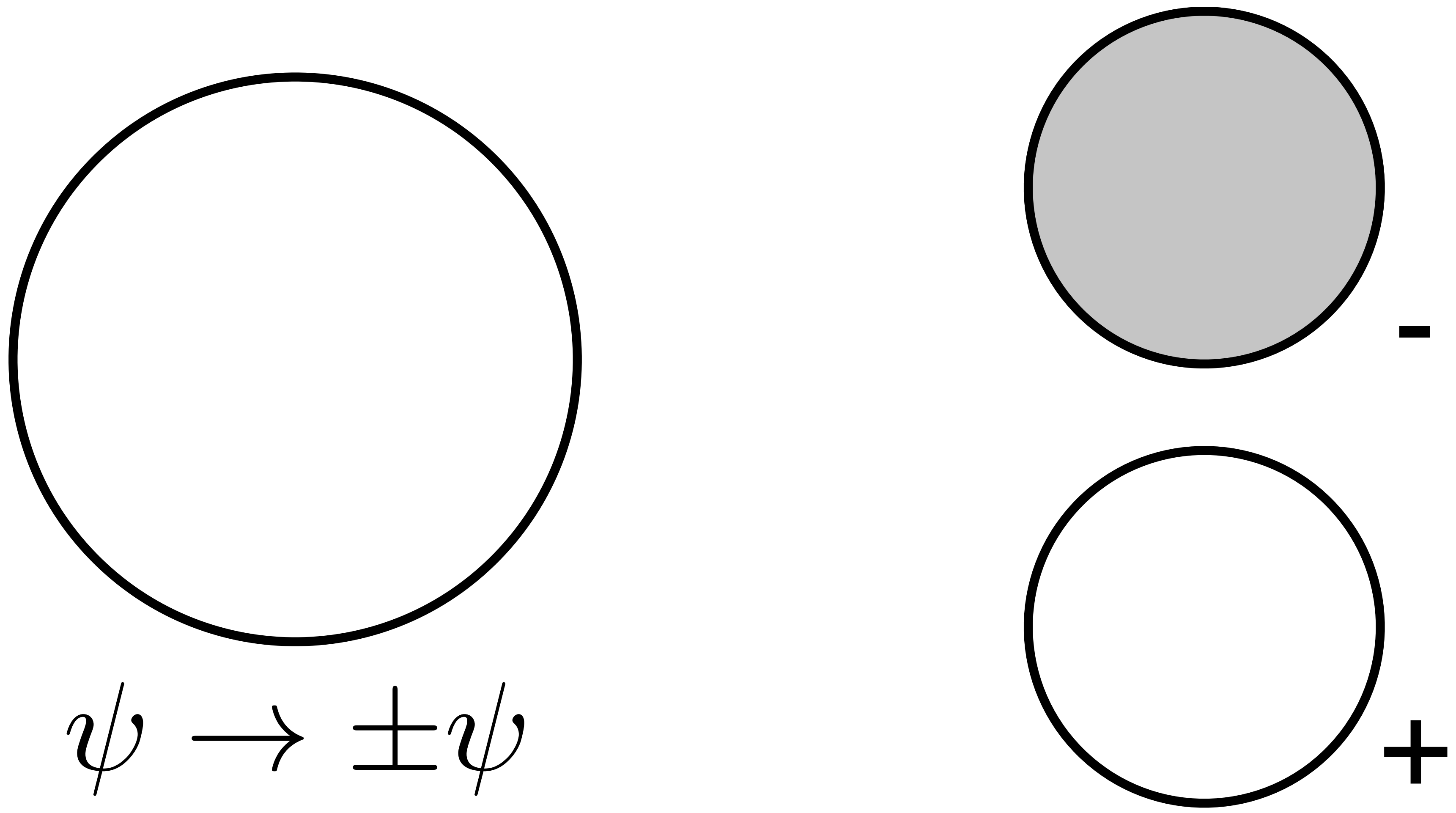
Does the structure matter?

Yes!



in $\Omega_1^{\mathrm{SO}}(pt)$

$$S^1 = \partial D^2$$



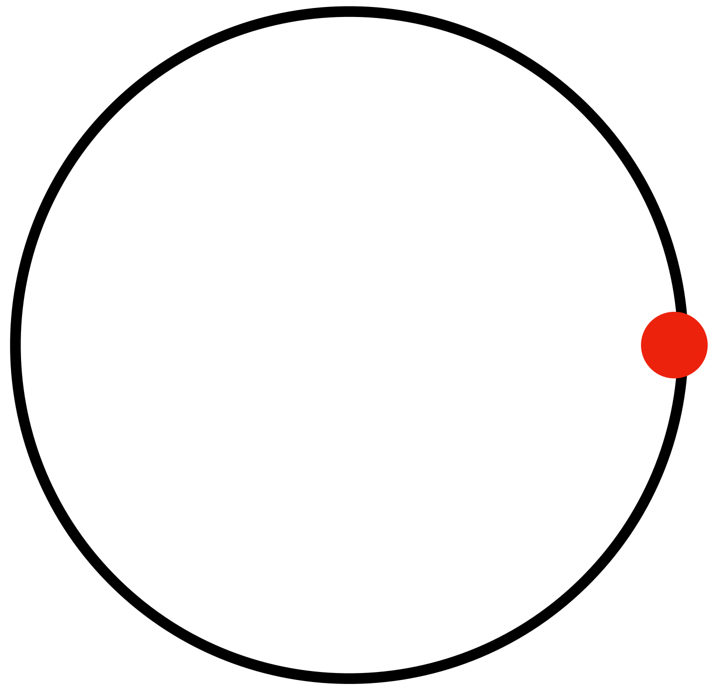
$$\psi \to \pm\psi$$

in $\Omega_1^{\mathrm{Spin}}(pt)$

choice of spin structure $H^1(S^1, \mathbb{Z}_2)$

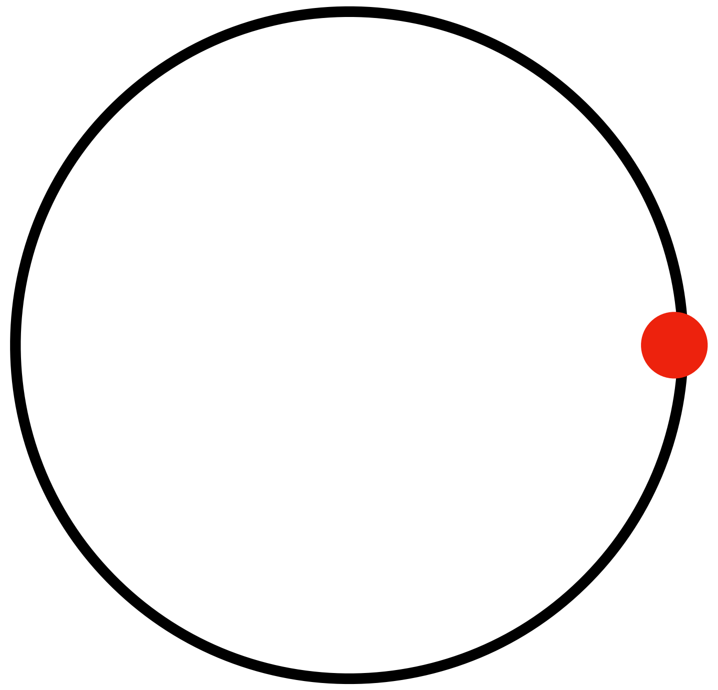
Does the gauge group matter?

Yes!

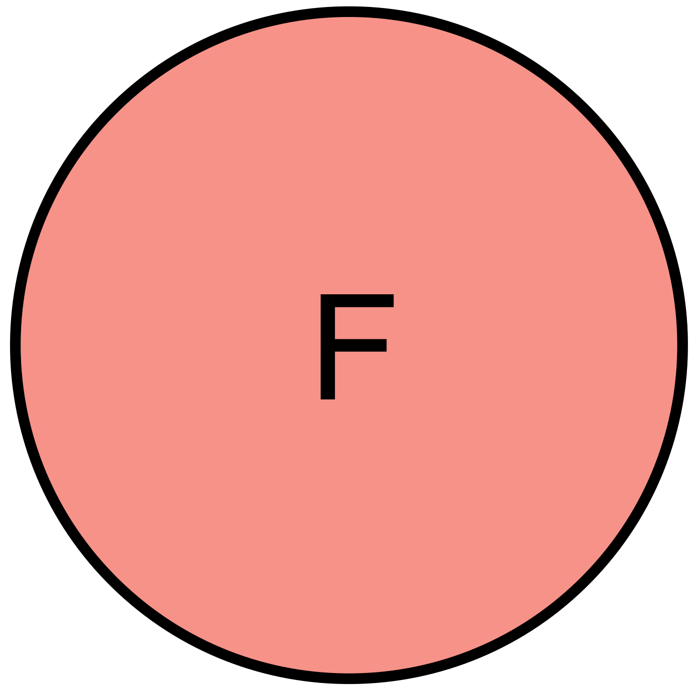


$g \in \mathbb{Z}_k$ in $\Omega_1^{\mathrm{SO}}(B\mathbb{Z}_k)$

choice of discrete gauge bundle $H^1(S^1, \mathbb{Z}_k)$



$g \in \mathbb{Z}_k \subset \mathrm{U}(1)$



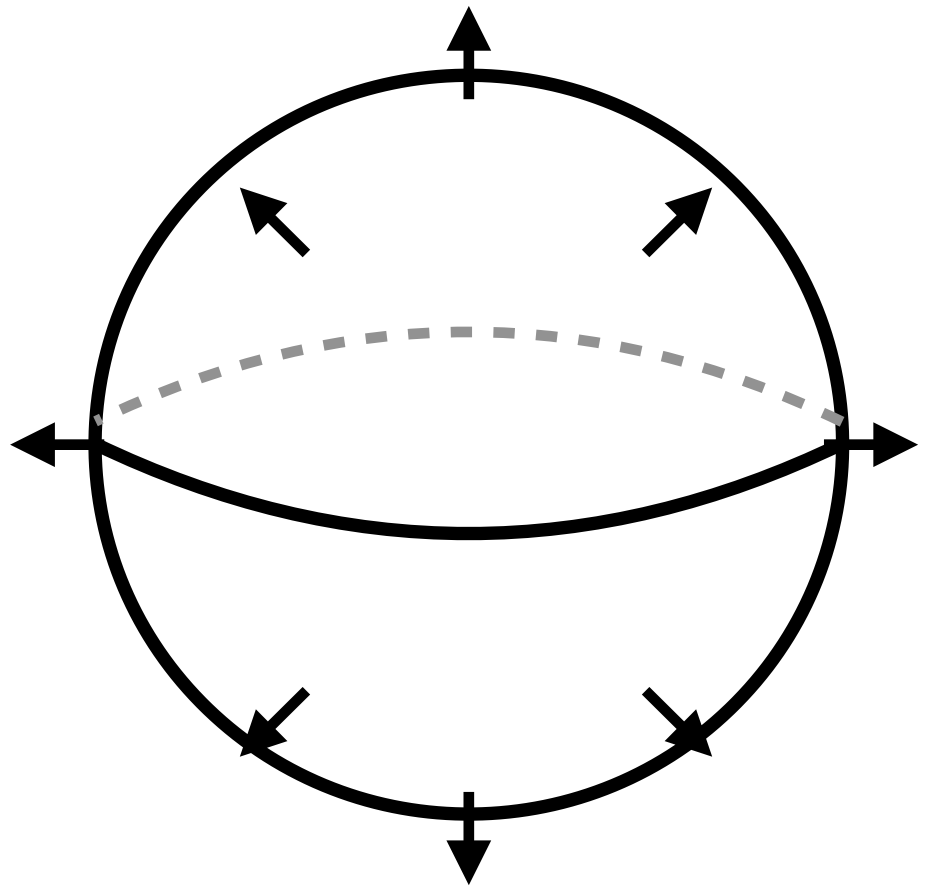
in $\Omega_1^{\mathrm{SO}}(B\mathrm{U}(1))$

And both matter at the same time

U(1) gauge field via classifying maps into:

$$BU(1) \simeq \mathbb{CP}^\infty$$

$$H^n\big(BU(1);\mathbb{Z}\big) = \mathbb{Z}[x]\,, \quad x \in H^2\big(BU(1);\mathbb{Z}\big)$$



Topological class specified by first Chern class:

$$c_1 \sim \frac{1}{2\pi}F \longrightarrow c_1 \cup c_1 \cup \cdots \cup c_1$$

but on Spin manifolds: $c_1 \cup c_1 \mod 2 = c_1 \cup w_2 = 0$ (false on oriented)

How does one compute?

Spectral sequences I: Atiyah-Hirzebruch (for generalized cohom)

[Atiyah, Hirzebruch ’61]

(Serre) fibration: $F \hookrightarrow X \to B$ (for us mainly $F = \mathrm{pt}$)

$$E^2_{p,q} = H_p\big(B, \Omega^{\mathrm{Spin}}_q(F)\big) \longrightarrow E^\infty_{p,q} \Rightarrow \Omega^{\mathrm{Spin}}_{p+q}(X)$$

$\Omega^{\mathrm{Spin}}_k\big(B\mathrm{U}(1)\big)$

$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	0	0	0
$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$
$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$
$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$

k	bordism group
0	$\mathbb{Z}$
1	$\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}$
3	0
4	$\mathbb{Z} \oplus \mathbb{Z}$

How does one compute?

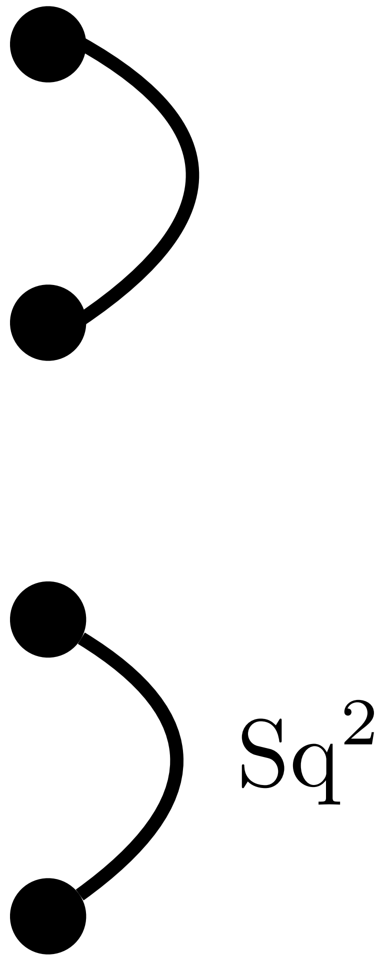
Spectral sequences II: Adams (homotopy of spectra at primes)

[Adams '58]

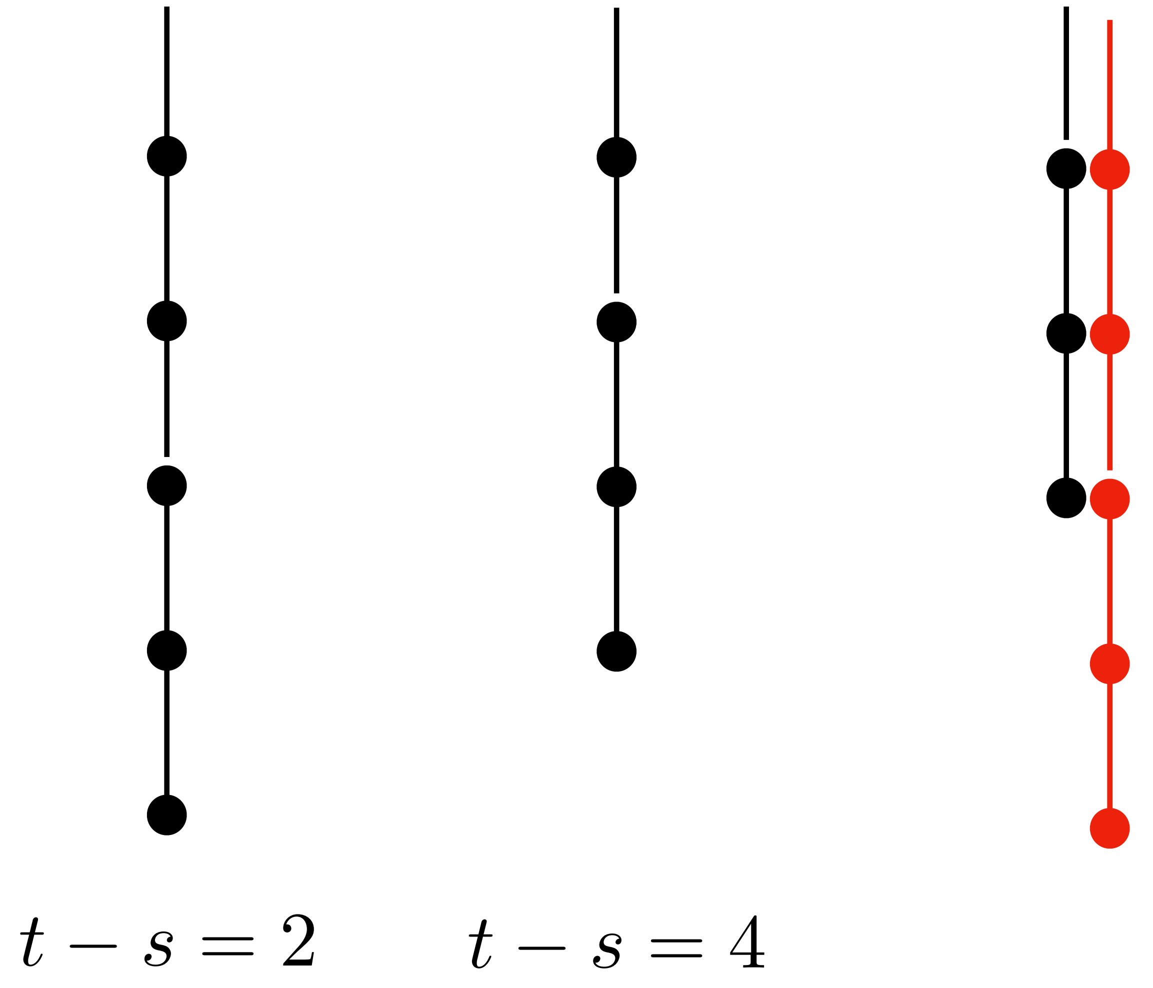
Each generalized (co)homology associated to a spectrum, e.g. $M\mathrm{Spin}$

$$E_2^{s,t} = \mathrm{Ext}_{\mathcal{A}}^{s,t}\big(\widetilde{H}^\bullet(M\mathrm{Spin} \wedge X; \mathbb{Z}); \mathbb{Z}_2\big) \Rightarrow \pi_{t-s}^{\mathrm{st}}(M\mathrm{Spin} \wedge X)_2^\wedge \simeq \widetilde{\Omega}_{t-s}^{\mathrm{Spin}}(X)_2^\wedge$$

$$\widetilde{H}^\bullet\big(BU(1); \mathbb{Z}_2\big)$$



Adams chart



$$\Omega_2^{\mathrm{Spin}}\big(BU(1)\big) \supset \mathbb{Z}$$

$$\Omega_4^{\mathrm{Spin}}\big(BU(1)\big) \supset \mathbb{Z}$$

$$\Omega_6^{\mathrm{Spin}}\big(BU(1)\big) \supset \mathbb{Z} \oplus \mathbb{Z}$$

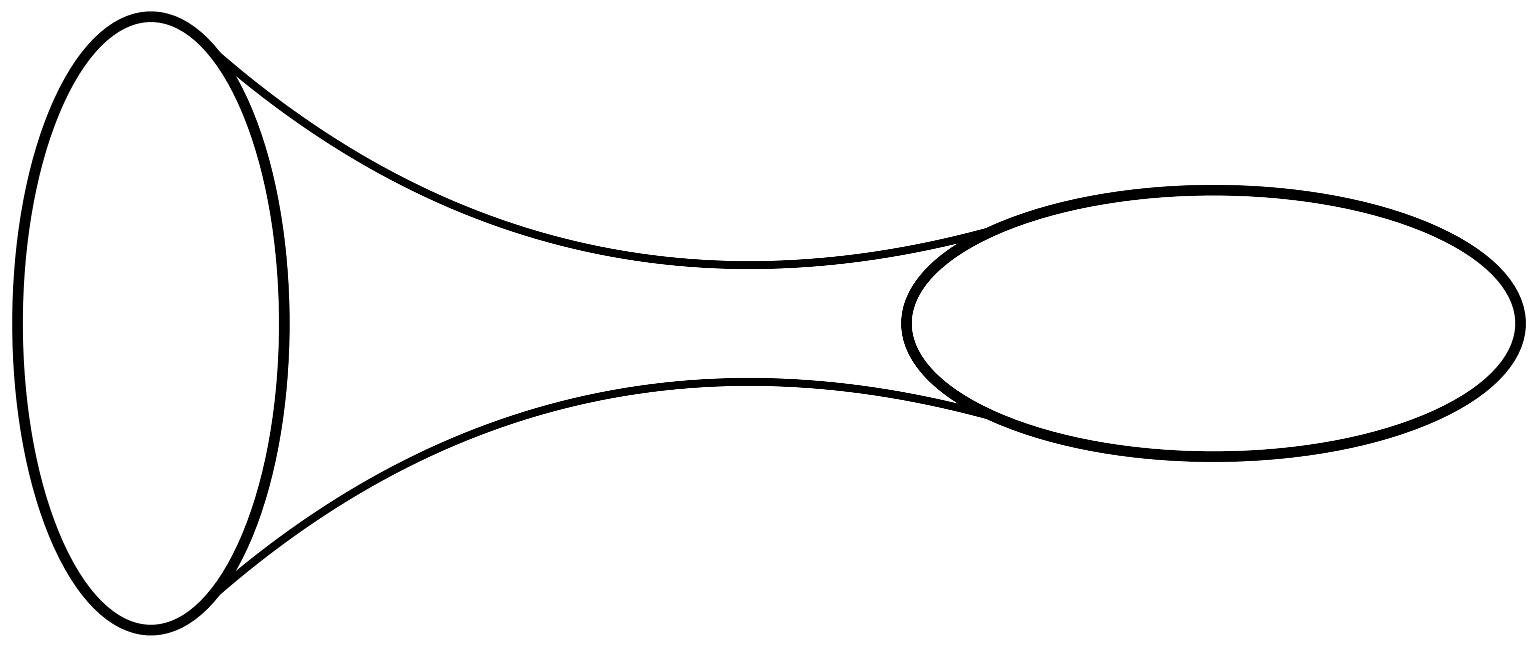
Bordisms in Quantum Gravity

Quantum Gravity

(Here we work in Euclidean signature)

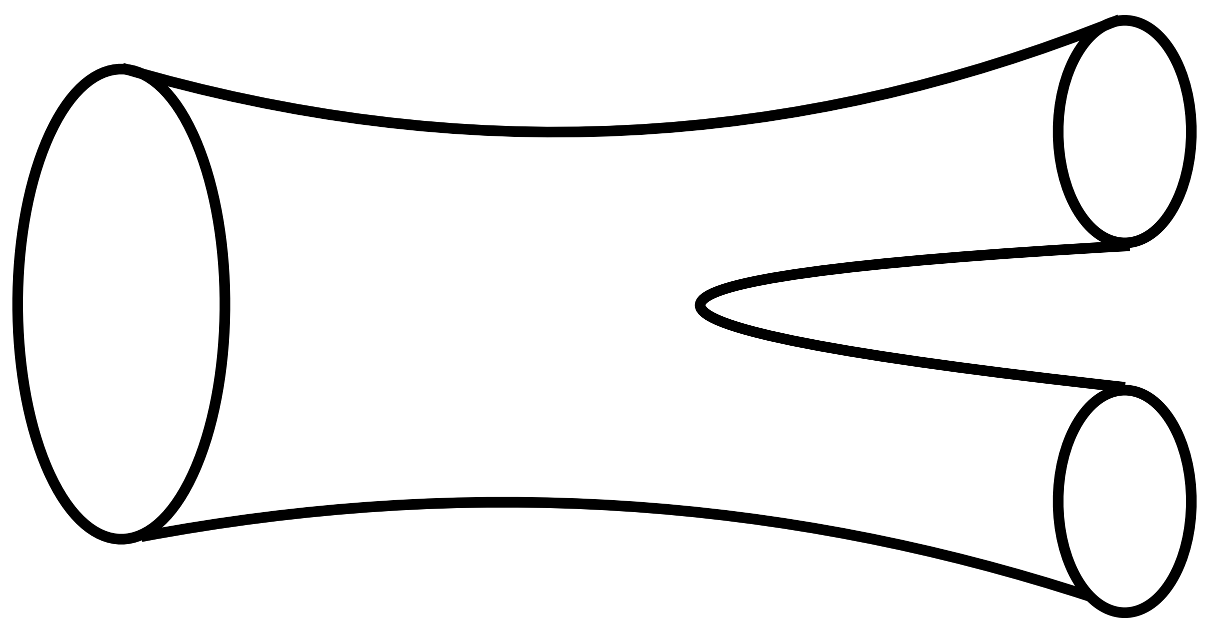
Gravity: dynamics of spacetime ✅

(not so interesting from bordism perspective;
continuous deformation provides bordism)



Quantum Gravity: changes of topology of spacetime ✅

(Way more interesting from bordism perspective)

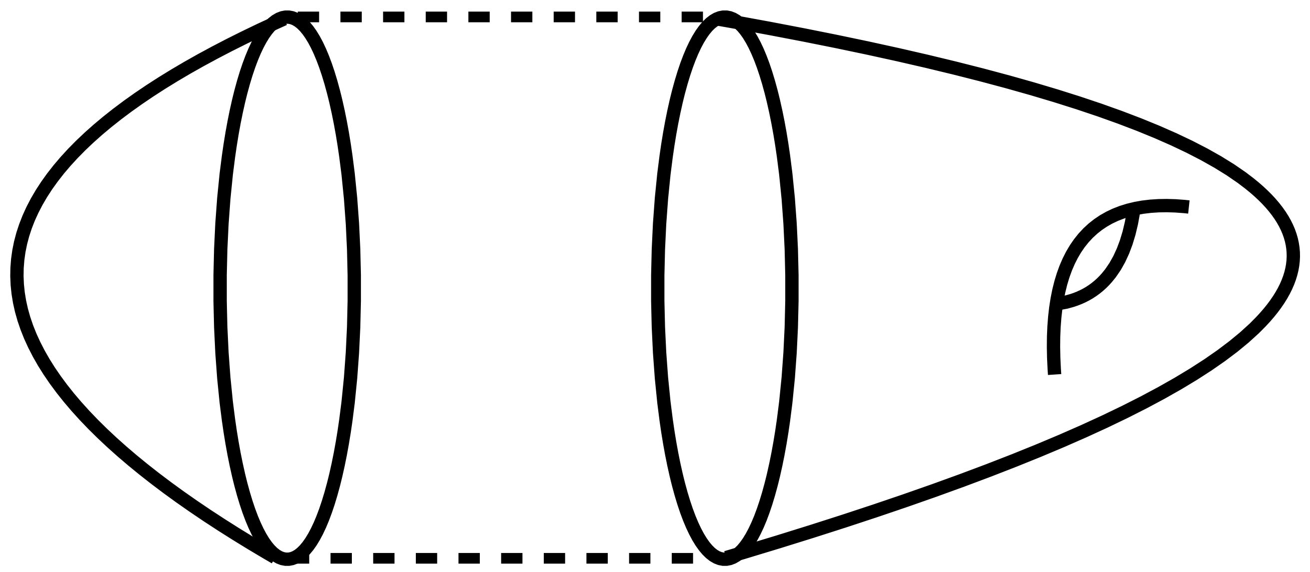


→ **Bordisms** capture some **topological features** of the **low-energy limit** of quantum gravity

Bordisms in Quantum Gravity

Anomalies

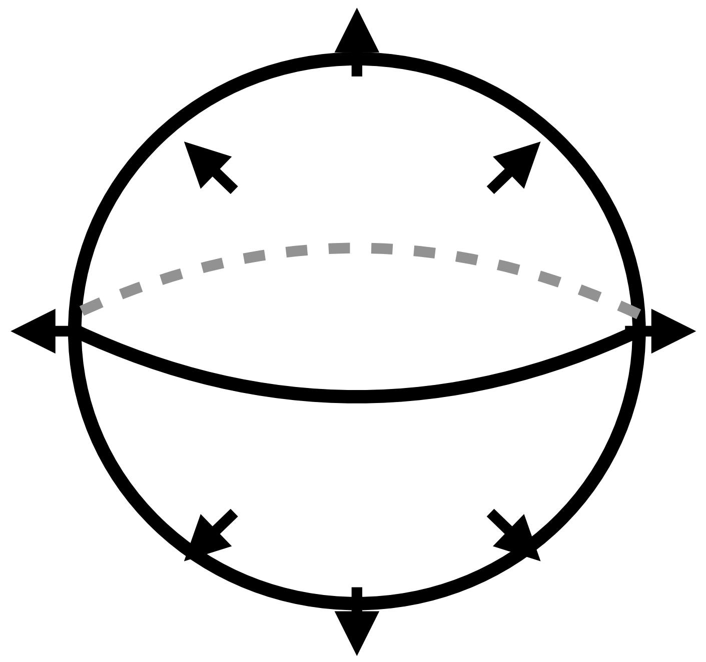
breaking of symmetries via quantum gravity effects



inconsistency for gauge symmetries

Global symmetries

definition of conserved charges associated to symmetries



absent in quantum gravity theories

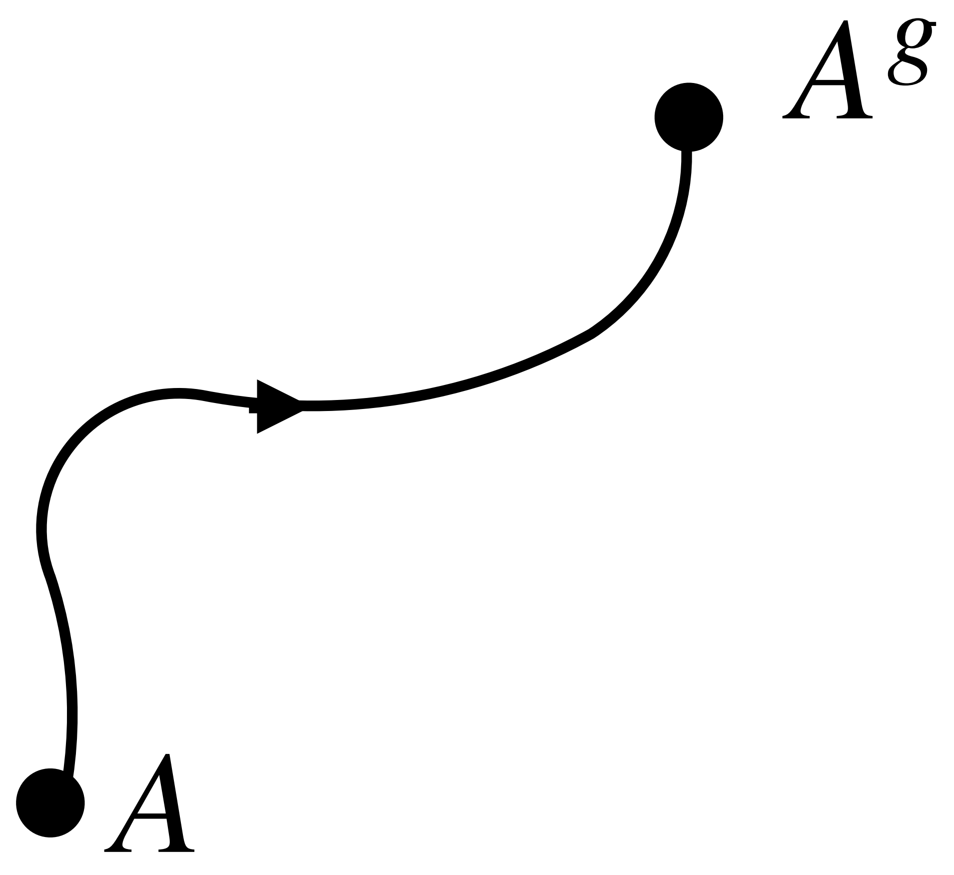
Bordisms and anomalies

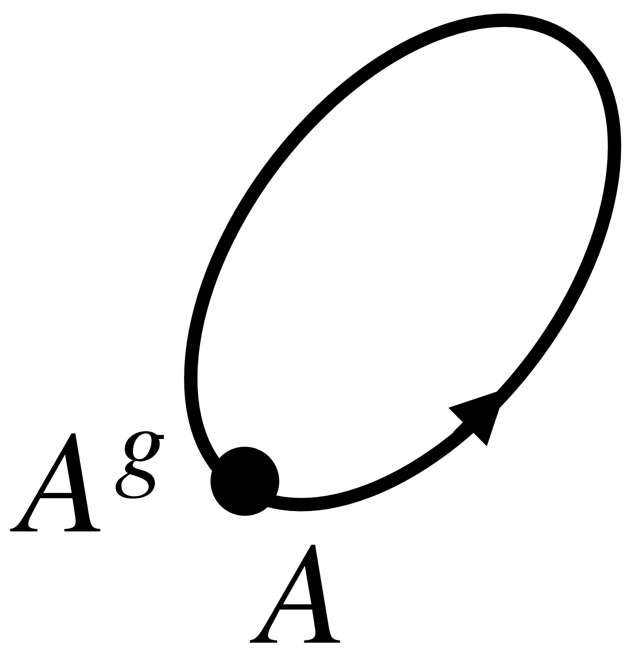
Anomalies

- **Couple symmetry to background connection** A
- **Move in configuration space** $A \to A^g$
- **Calculate partition function**

$$Z[A] \neq Z[A^g]$$

$$Z[A^g] = e^{i\alpha} Z[A]$$



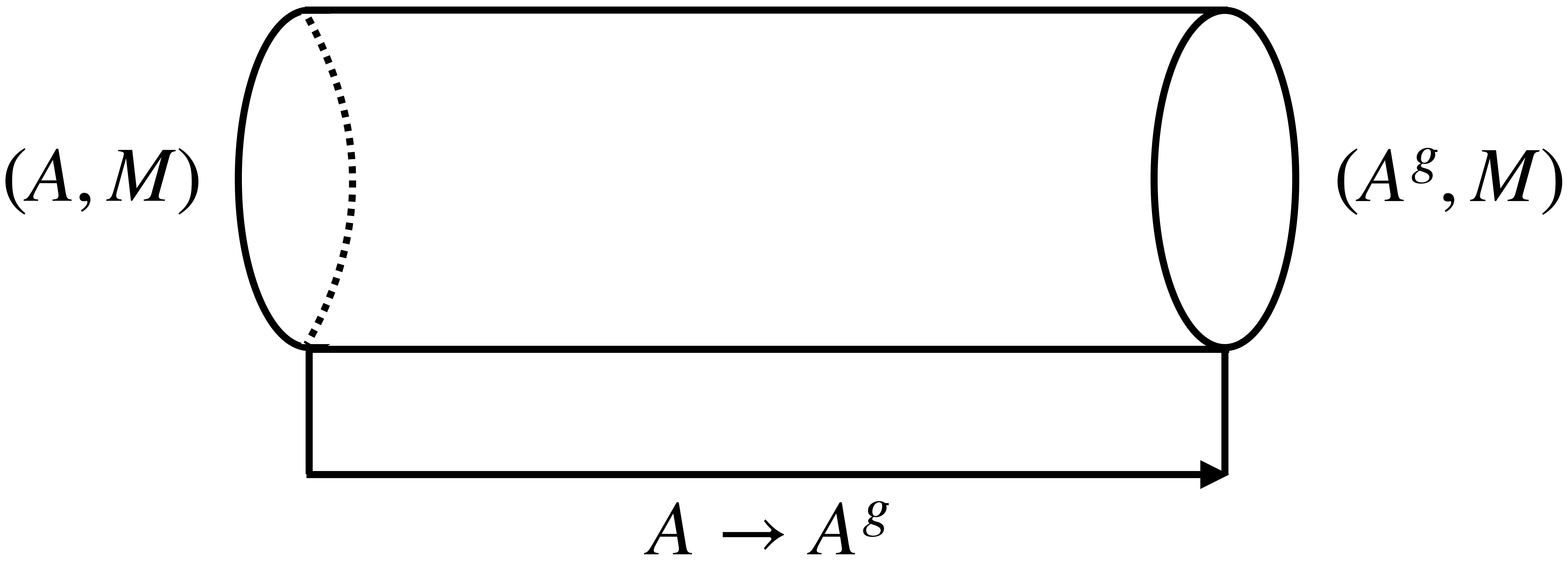


Perturbative anomalies

[Adler '69, Bell, Jackiw '69]

- **Small variations** (contractible paths): **perturbative anomalies**
- **Symmetry** needs to be **continuous**

Geometrize

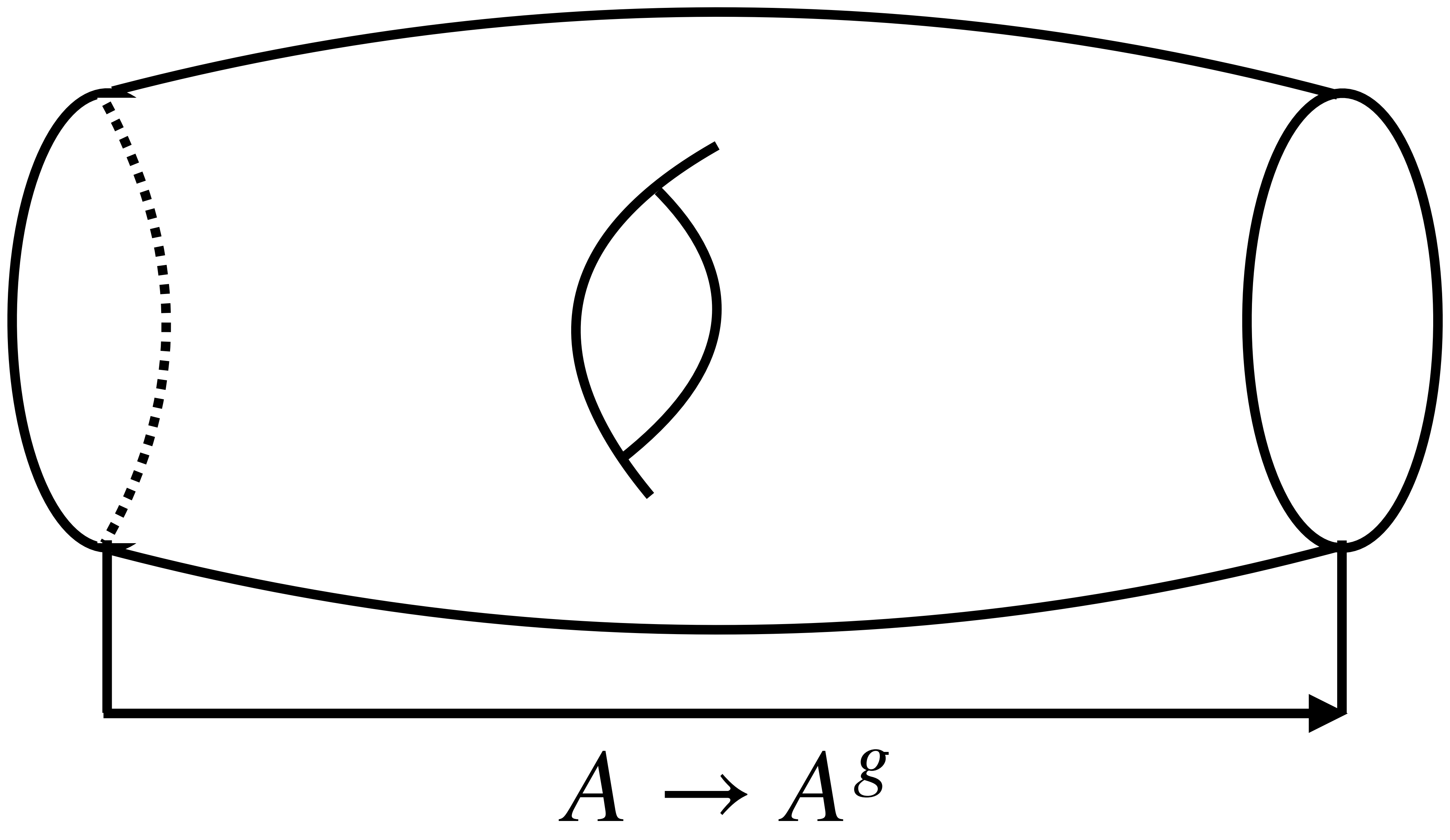


- **Large variations** (non-contractible paths): **global anomalies**
- Spans a **(d+1)-dimensional manifold**

➡ gluing the ends: **mapping tori** [Witten '82]

Dai-Freed anomaly

[Dai, Freed '94], [Witten '15], [Yonekura '16], see also [Montero, Garcia-Etxebarria '18] for a great review



- **Topology changes along path ➡ 'quantum gravity' flavor**
- Forms **(d+1)-dimensional manifold** with given structure
- **Detected** by evaluation of **(d+1)-dimensional anomaly theory**

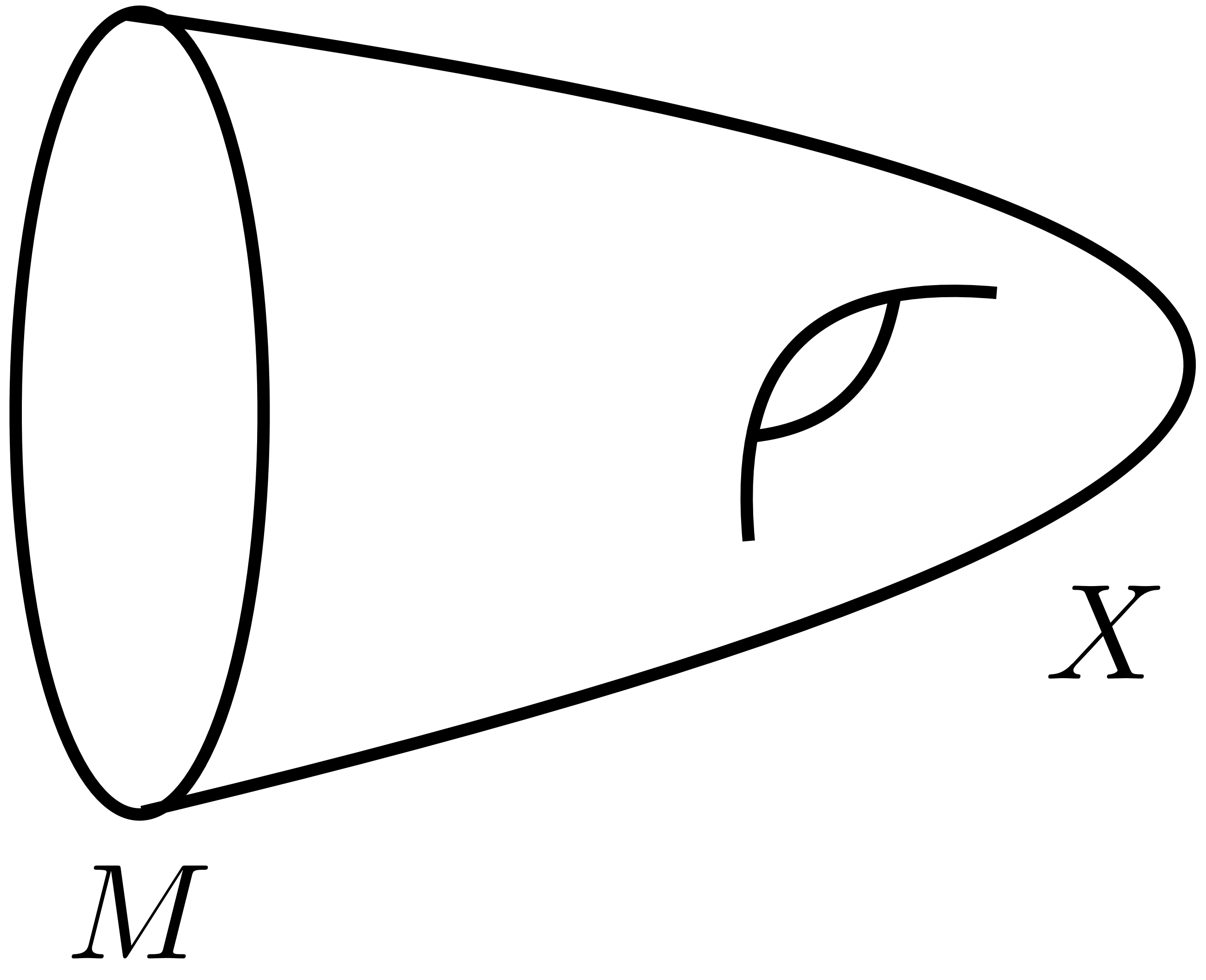
Anomaly field theory

e.g. [Freed, Teleman '14]

one-dimensional Hilbert space: only phase

There is a (d+1)-dimensional invertible field theory $\mathcal{A}$ such that:

$$\frac{Z[M]}{|Z[M]|} = e^{2\pi i \mathcal{A}[X]}, \quad \partial X = M$$

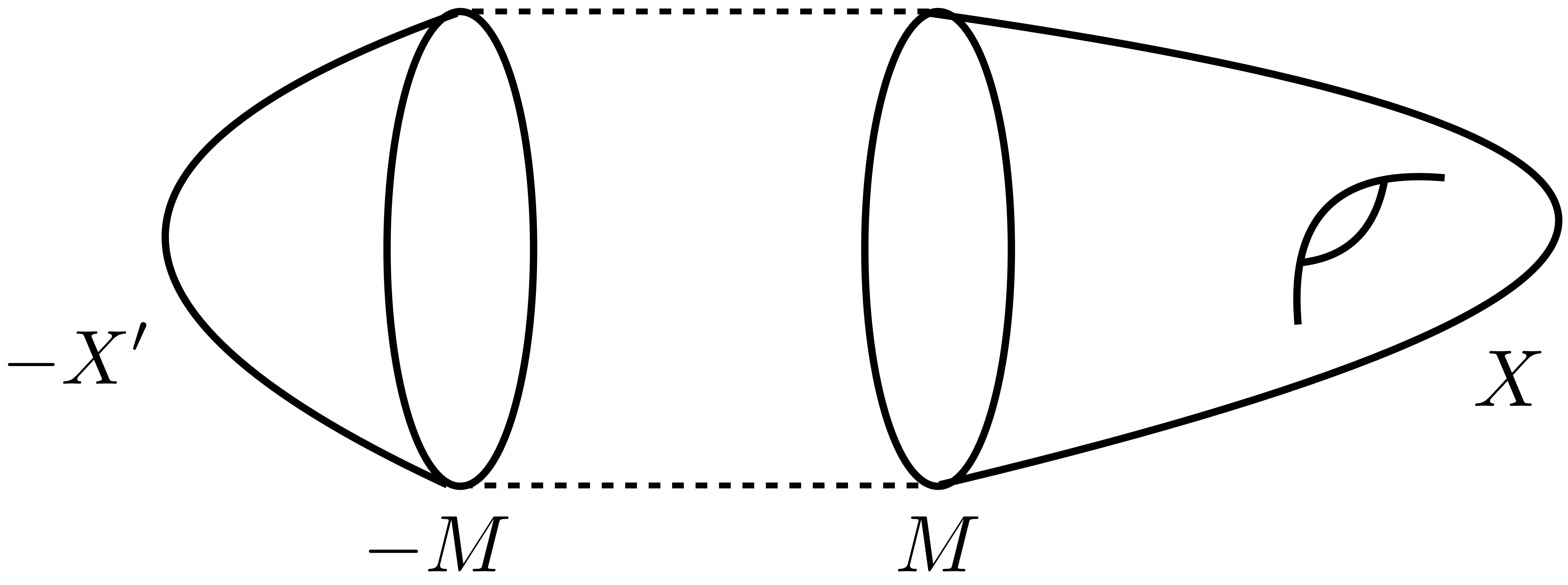


Boundary theory determines structure:

- Spin structure
- Gauge fields
- …

No Dai-Freed anomalies ⟷ Independent of extension

Anomaly field theory



$$\mathcal{A}[X] = \mathcal{A}[X'] \quad \rightarrow \quad \boxed{e^{2\pi i \mathcal{A}[Y]} = 1}$$

for **all closed manifolds** Y with wanted physical **structure**

→ classified by **bordism groups**

Bordisms and anomaly

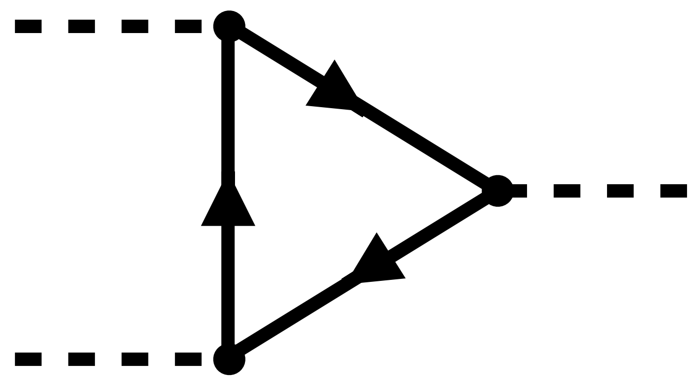
more precisely there is a **short exact sequence**: (Anderson dual)

$$0 \longrightarrow \mathrm{Ext}\big(\Omega^{\mathrm{Spin}}_{d+1}(BG);\mathbb{Z}\big) \longrightarrow \big(I_{\mathbb{Z}}\Omega^{\mathrm{Spin}}\big)^{d+2}(BG) \longrightarrow \mathrm{Hom}\big(\Omega^{\mathrm{Spin}}_{d+2}(BG);\mathbb{Z}\big) \longrightarrow 0$$

Morally:

- $\mathrm{Tors}\big(\Omega^{\mathrm{Spin}}_{d+1}(BG)\big)$ classifies **perturbative anomalies**
- $\mathrm{Free}\big(\Omega^{\mathrm{Spin}}_{d+2}(BG)\big)$ classifies **non-perturbative anomalies**

Example U(1) in 4d: $\Omega^{\mathrm{Spin}}_5\big(BU(1)\big) = 0\,,\;\; \Omega^{\mathrm{Spin}}_6\big(BU(1)\big) = \mathbb{Z}\oplus\mathbb{Z}$



$F^3\,,\quad F\,\mathrm{tr}R^2$

Anomalies: Strategy

- **Determine structure** ξ (from physical theory)
- **Find the anomaly theory** $\mathcal{A}$
- **Determine bordism groups** $\Omega^{\xi}_{d+1}, \Omega^{\xi}_{d+2}$
- **Find a set of generators for bordism group** (highly non-trivial)
- **Evaluate anomaly theory on the set of generators**

→ **No anomalies if:** $\mathcal{A}[X] \in \mathbb{Z}$

Example: 4d U(1) theory

Fermions with arbitrary integer charges ➞

$$\Omega_5^{\mathrm{Spin}}(B\mathrm{U}(1)) = 0$$

$$\Omega_6^{\mathrm{Spin}}(B\mathrm{U}(1)) = \mathbb{Z} \oplus \mathbb{Z}$$

Anomaly theory: $\mathcal{A} = \sum_F \pm \eta_q^{\mathrm{D}}$

Using the APS index theorem: $\int \sum_F \pm \hat{A}(R)\mathrm{ch}(qc_1)$

[Atiyah, Patodi, Singer '75]

(generators here not so important)

Expansion leads to anomaly conditions:

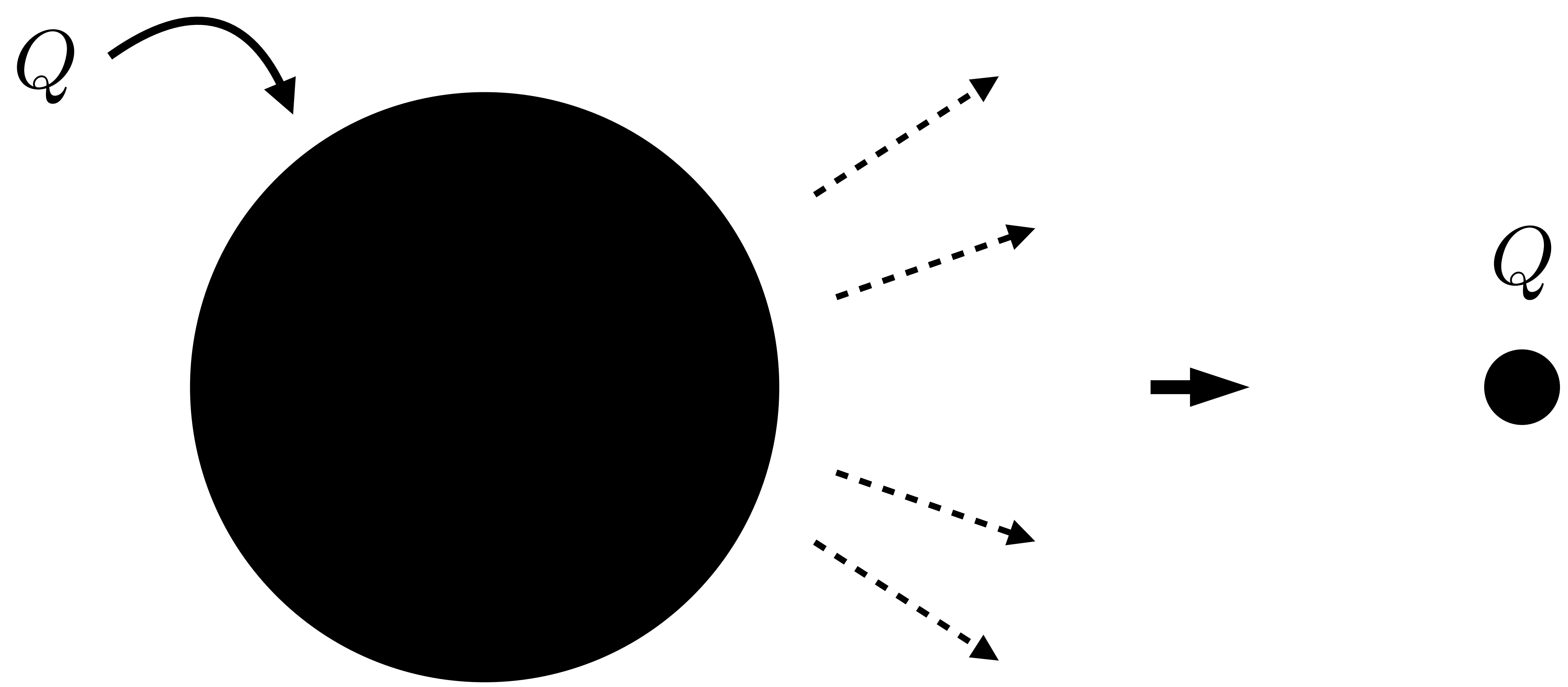
$$\sum_F \pm q^3 = 0 \qquad \sum_F \pm q = 0$$

Bordisms and global charges

Evidence for: No global symmetries

See e.g. [Banks, Dixon '88; Banks, Seiberg '11]

Global charge labels different states but **costs no energy**



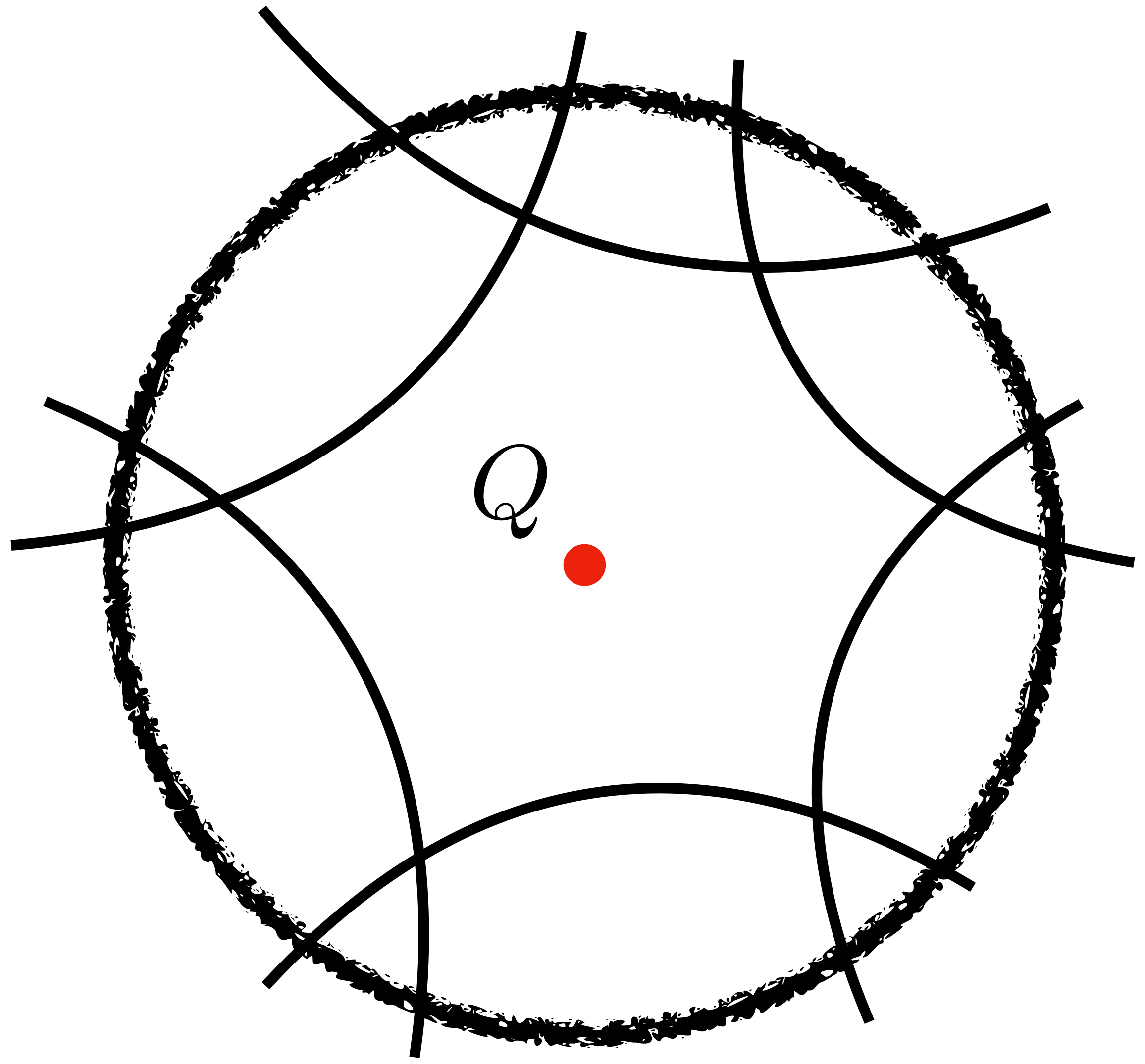
Remnants with arbitrary global charge (includes generalized symmetry charges)

↓

Violation of **entropy bounds**

Evidence for: No global symmetries

Remember: **Global charge labels different states** but **costs no energy**



Also leads to violation of
Holographic principle
[Harlow, Ooguri '18]

& violation in AdS/CFT
[Harlow, Shaghoulian '20],
[Bah, Chen, Maldacena '22],…

Violation typically **at least** $e^{-M_{Pl}^2/\Lambda^2}$

see also [Daus, Hebecker, Leonhardt, March-Russell '20]
for connection with weak gravity conjecture

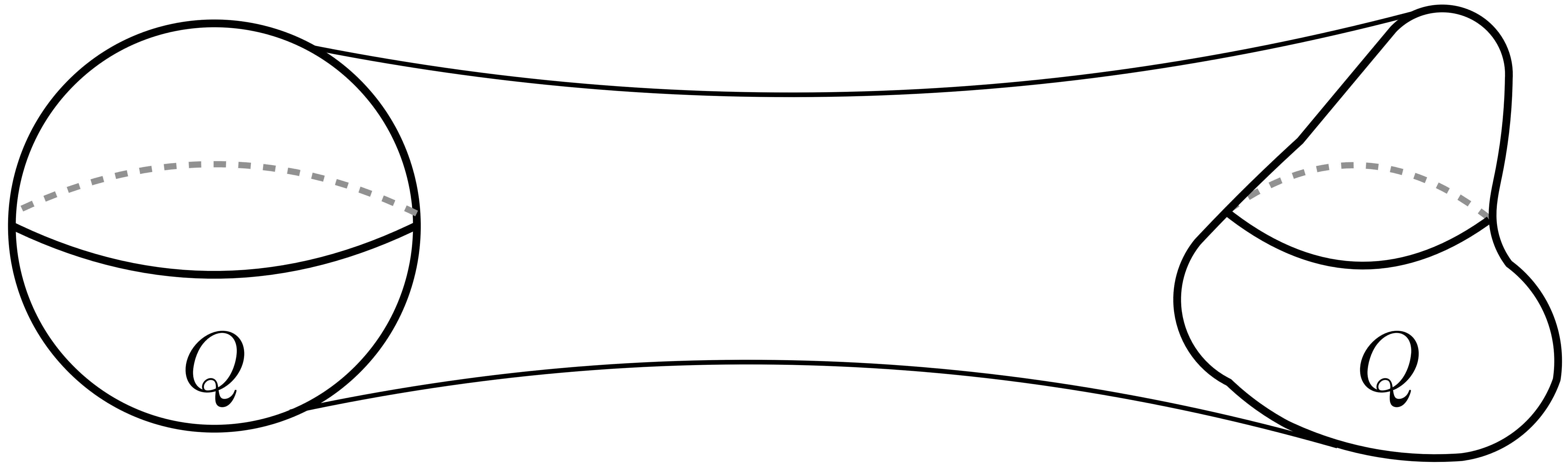
Non-perturbative
in **gravity**

Global symmetries

How can one be sure that **there are no global symmetries?**

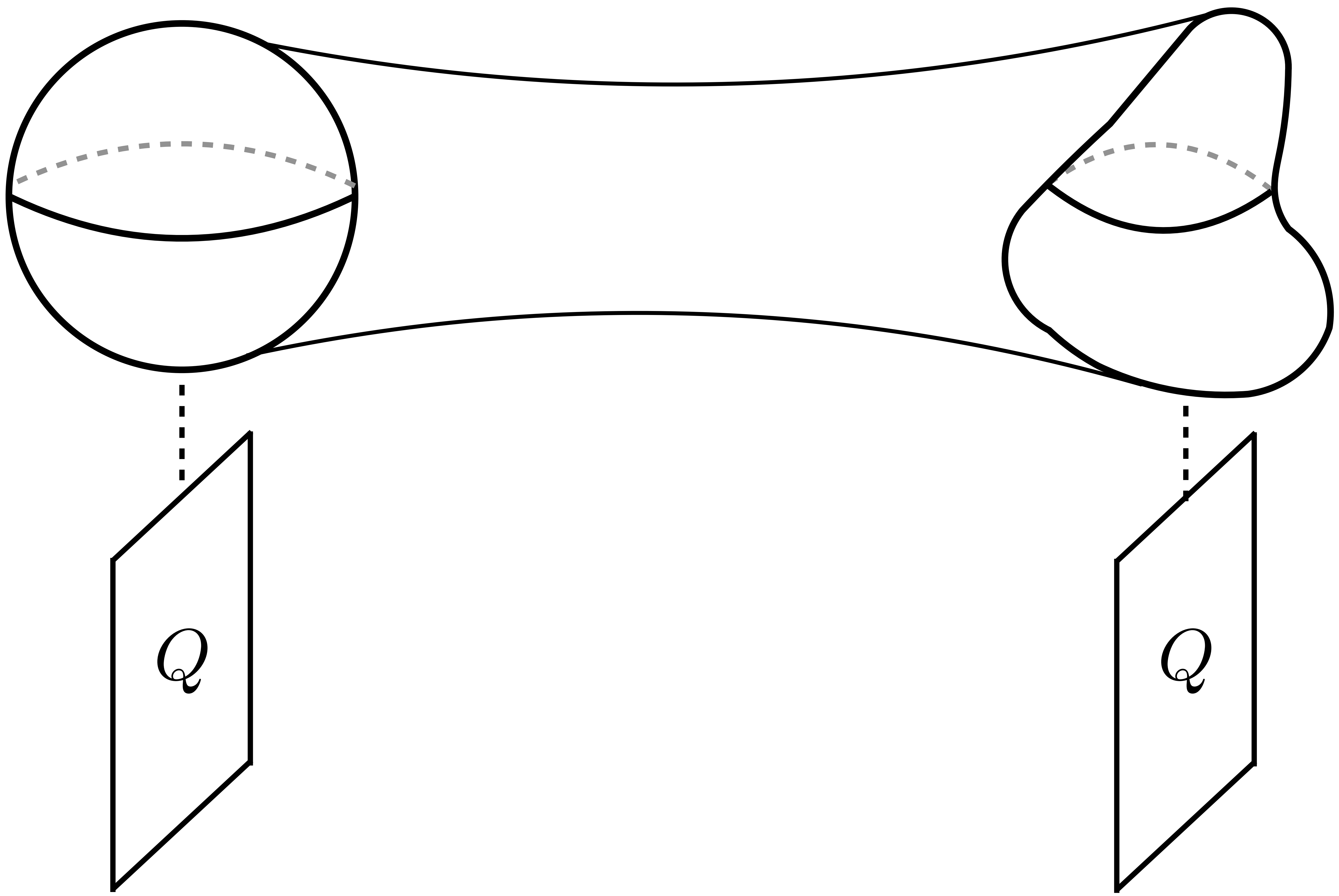
(no Lagrangian description, non-perturbative effects, …)

Global symmetries ⟷ Conserved charge



Global symmetries

Think of it as carried by **compactification space** (topologically)



at this stage no requirement to solve the equations of motion since the considered charges are **topological**

Global symmetries

How can one be sure that **there are no global symmetries?**

(no Lagrangian description, non-perturbative effects, …)

One state guaranteed to have **no charges:**

Global symmetries

How can one be sure that **there are no global symmetries?**

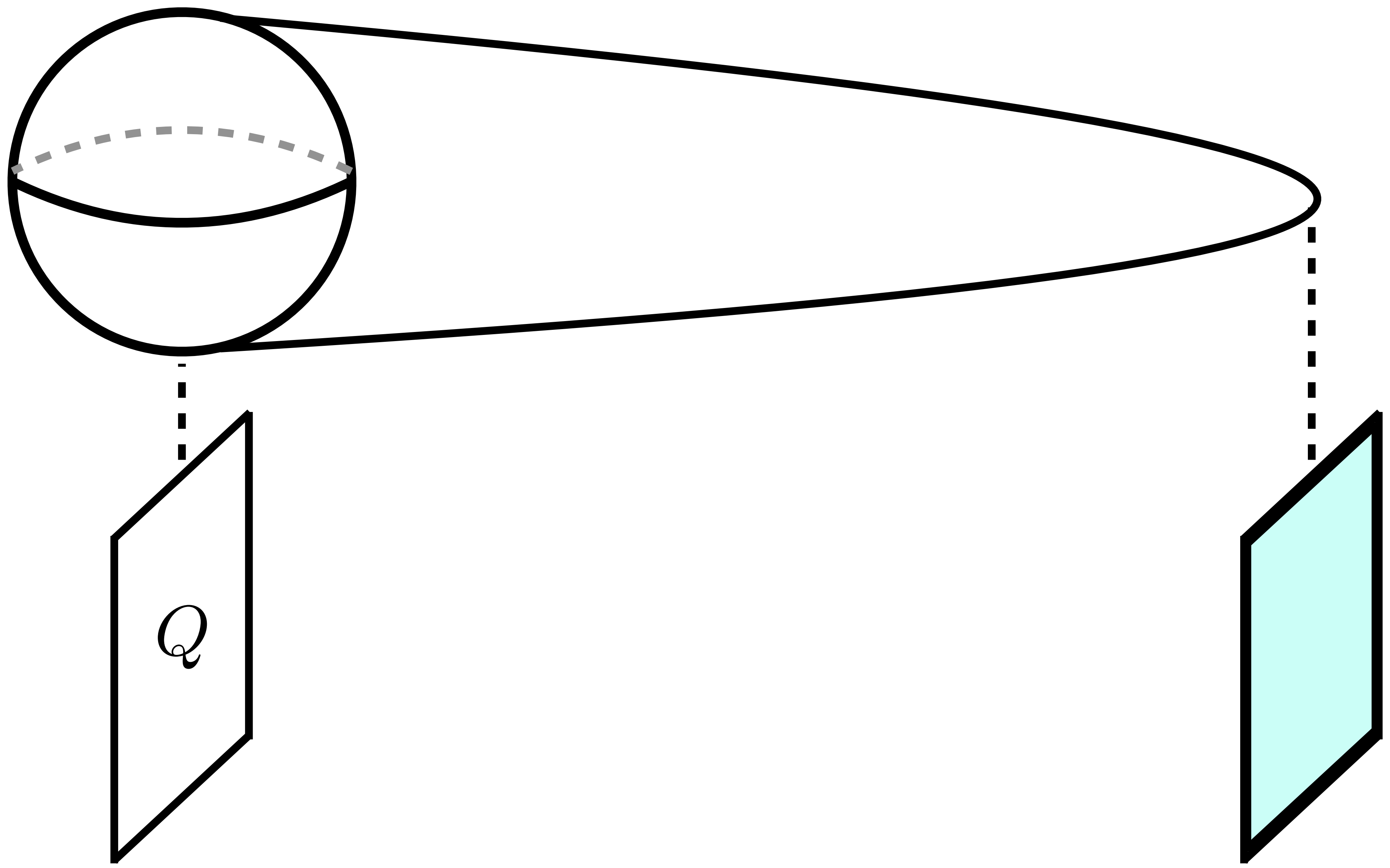
(no Lagrangian description, non-perturbative effects, …)

One state guaranteed to have **no charges:**

$$\varnothing$$

Global symmetries

If theory admits **transition to nothing** there are **no global symmetries**:



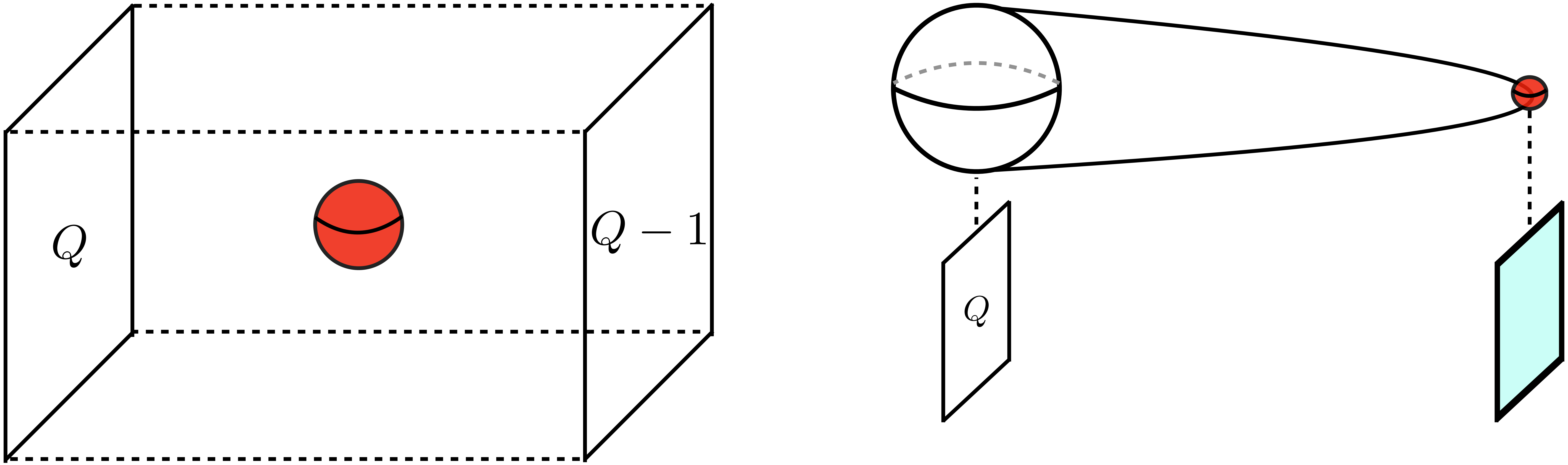
more mathematical: no non-trivial deformation classes captured by bordisms

$$\Omega_d^{\mathrm{QG}} = 0\,, \quad d < D$$ Swampland Cobordism Conjecture

[McNamara, Vafa ’19], also [Montero, Vafa ’20], [MD, Heckman ’20]

Symmetry-breaking objects

backgrounds associated with **new symmetry-breaking defects**

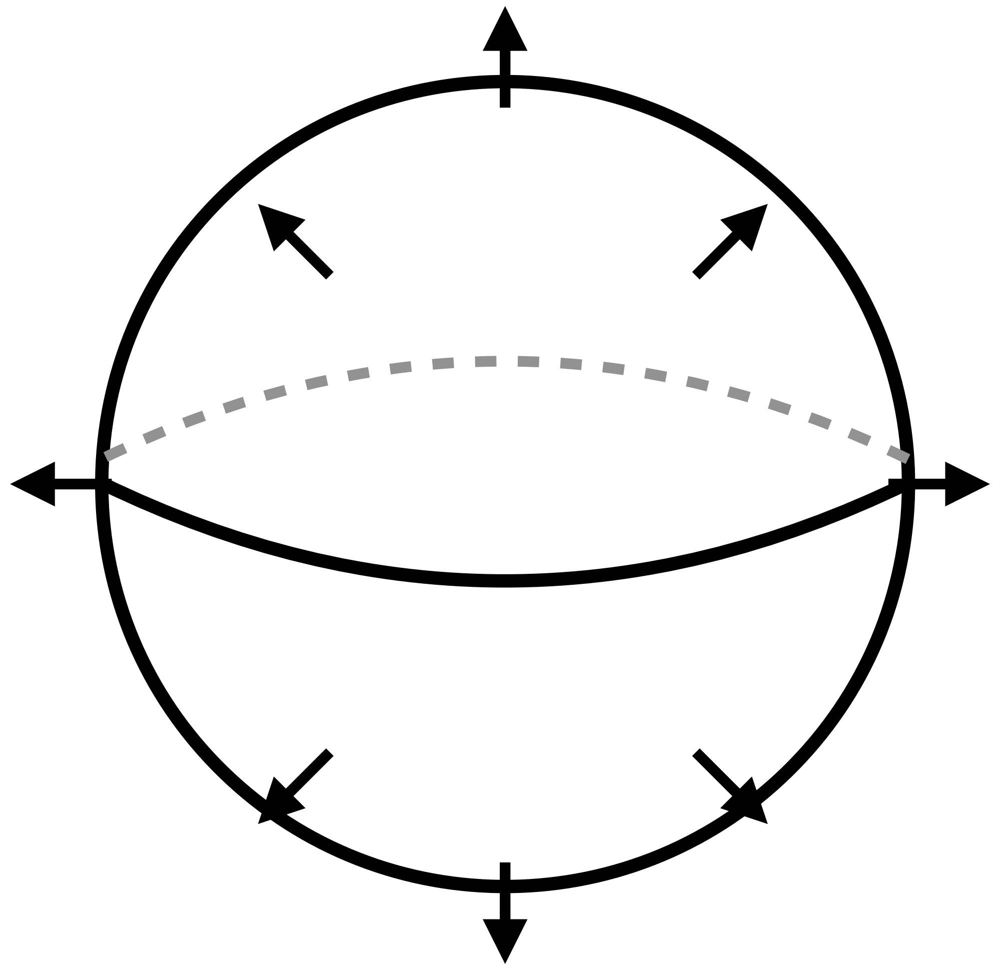


Can look **singular** at **low-energies**, but have **finite mass** (tension)

Global charges

Equivalently, the non-trivial deformation classes capture **global charges**

Example: U(1) gauge sector with fermions



$$\oint F \neq 0$$

threaded by magnetic flux

$$dF = 0$$

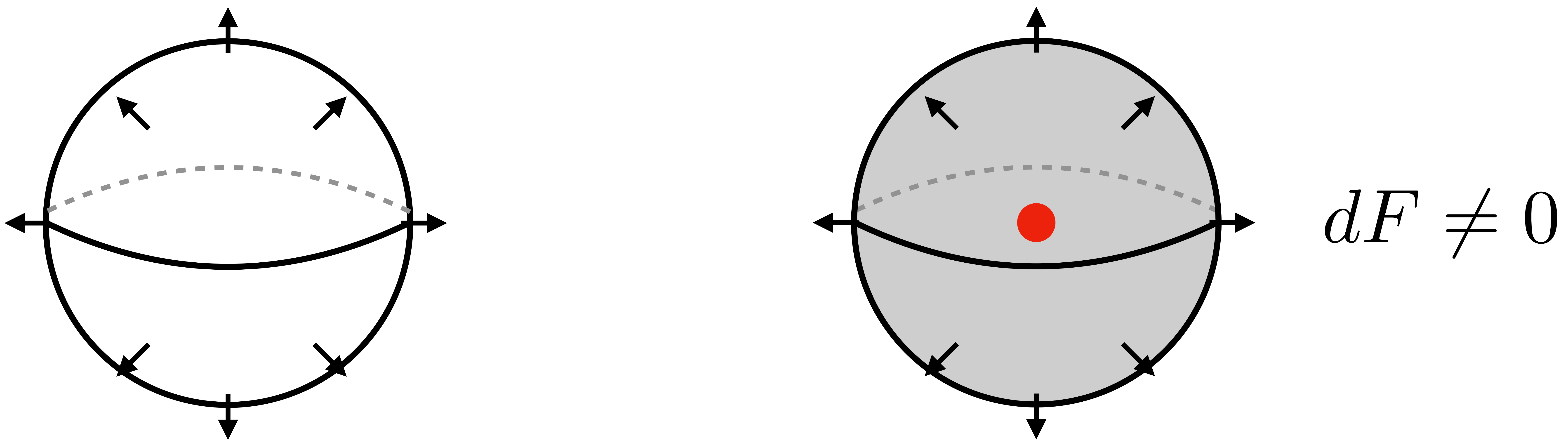
$$\not\to \ \varnothing$$

deformation classes of Spin manifolds with principal U(1) bundle

$$\Omega_2^{\mathrm{Spin}}(B\mathrm{U}(1)) \supset \mathbb{Z} \neq 0$$

What is the symmetry-breaking object?

Has the deformation class as **boundary**, can look **singular in IR**



This defect is a **magnetic monopole**; if **dynamical** it **breaks symmetry**

→ **predicted by quantum gravity** to avoid global symmetries

Similar conclusions for: axion strings, domain walls, …

Bordisms in Quantum Gravity: A test

More interesting example: IIBordia

Type IIB supergravity:

- properties of **spacetime** (tangential structure)
- **S-duality**: a gauged discrete strong-weak coupling duality

$$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}(\text{pt}) \qquad \Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}(\text{pt})$$

$$\begin{array}{ccc} \mathrm{SL}(2,\mathbb{Z}) & \longrightarrow & \mathrm{GL}(2,\mathbb{Z}) \\ \downarrow & & \downarrow \\ \mathrm{Mp}(2,\mathbb{Z}) & \longrightarrow & \mathrm{GL}^+(2,\mathbb{Z}) \end{array}$$

$\downarrow$ Spin/Pin lifts due to action on fermions

$\rightarrow$ orientation reversal of worldsheet

[Pantev, Sharpe '16], [Tachikawa, Yonekura '18]

What do we expect?

Remember (D = 10):

- $\Omega_{d<10} \neq 0$ lead to **global symmetries**
- $\Omega_{11} \neq 0$ leads to **potential global quantum gravity anomalies**

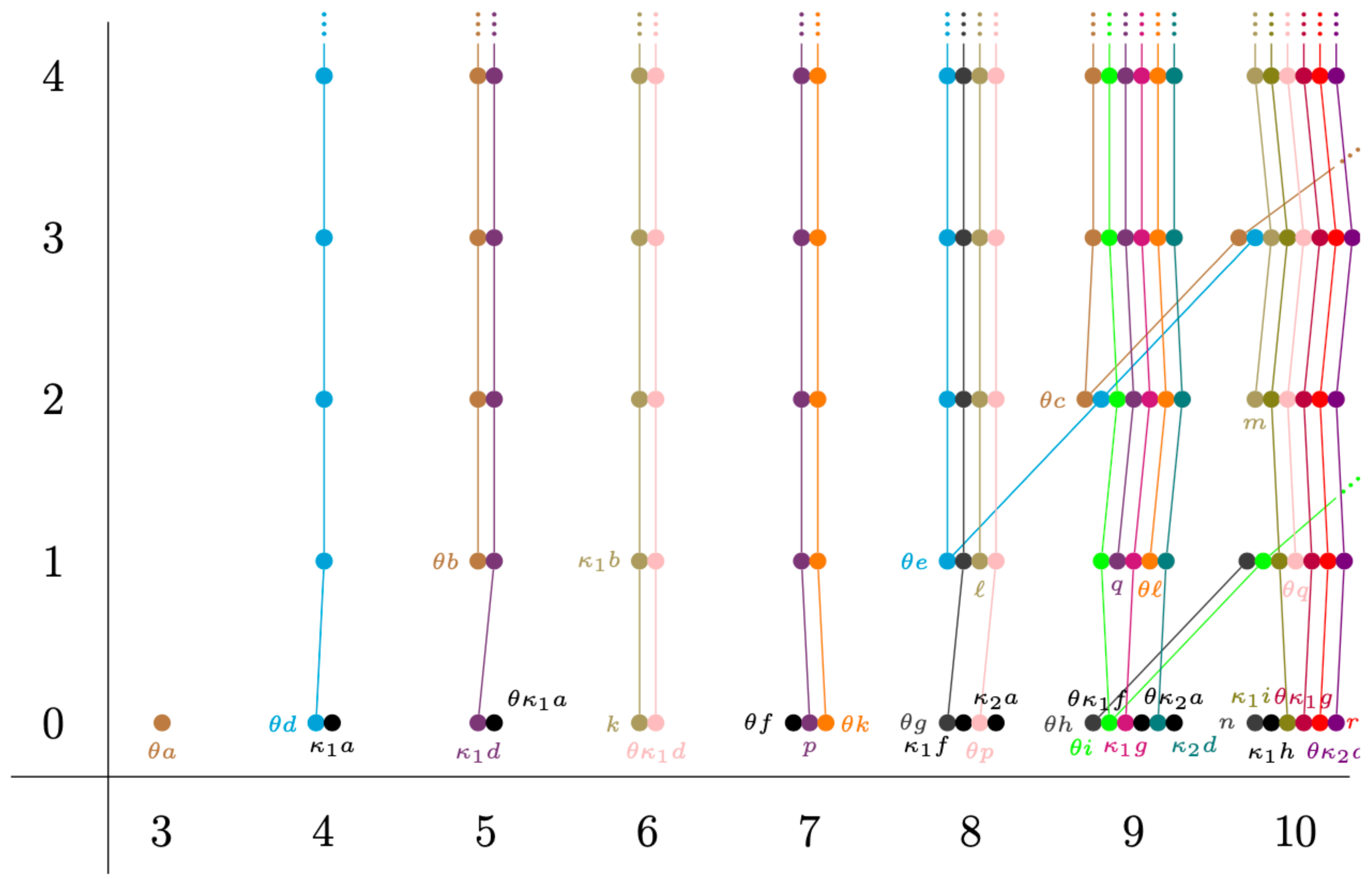
Since we know a consistent UV completion we expect:

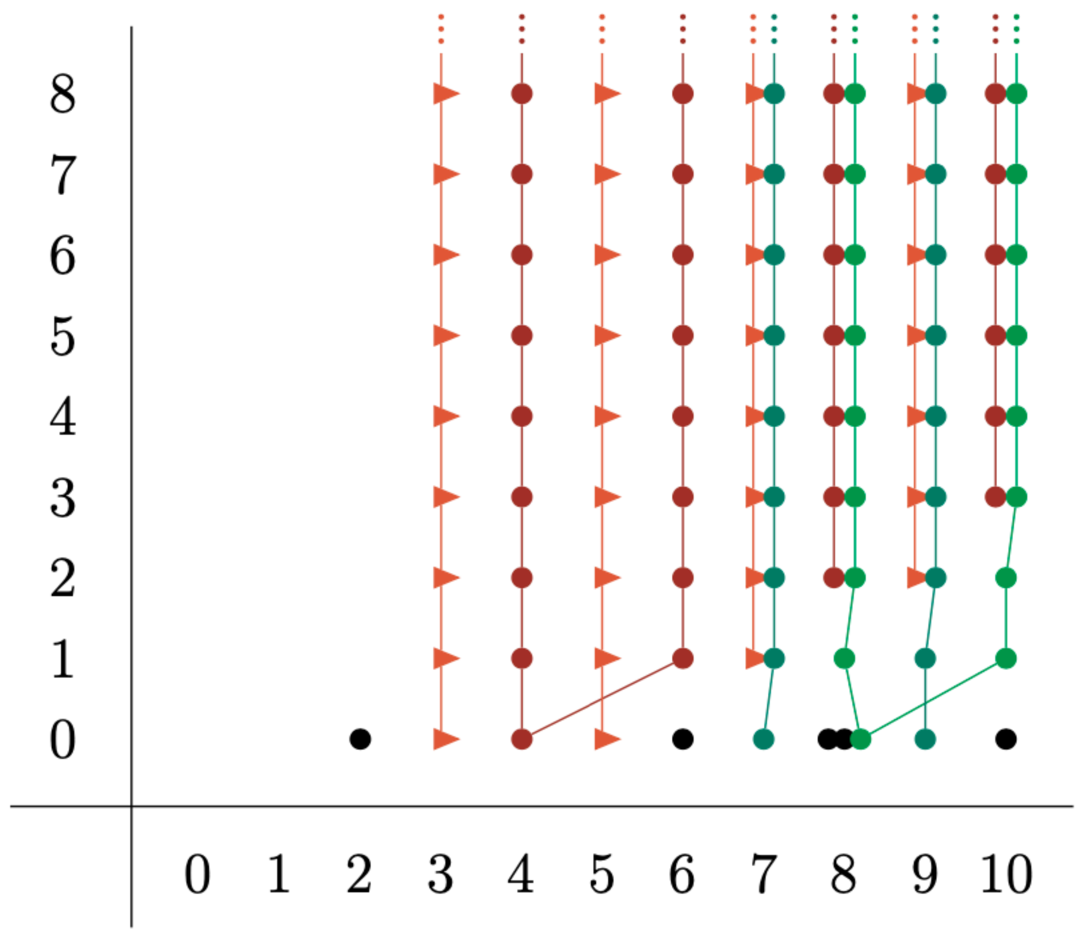
$$\Omega_d^{\mathrm{IIB}} = 0$$

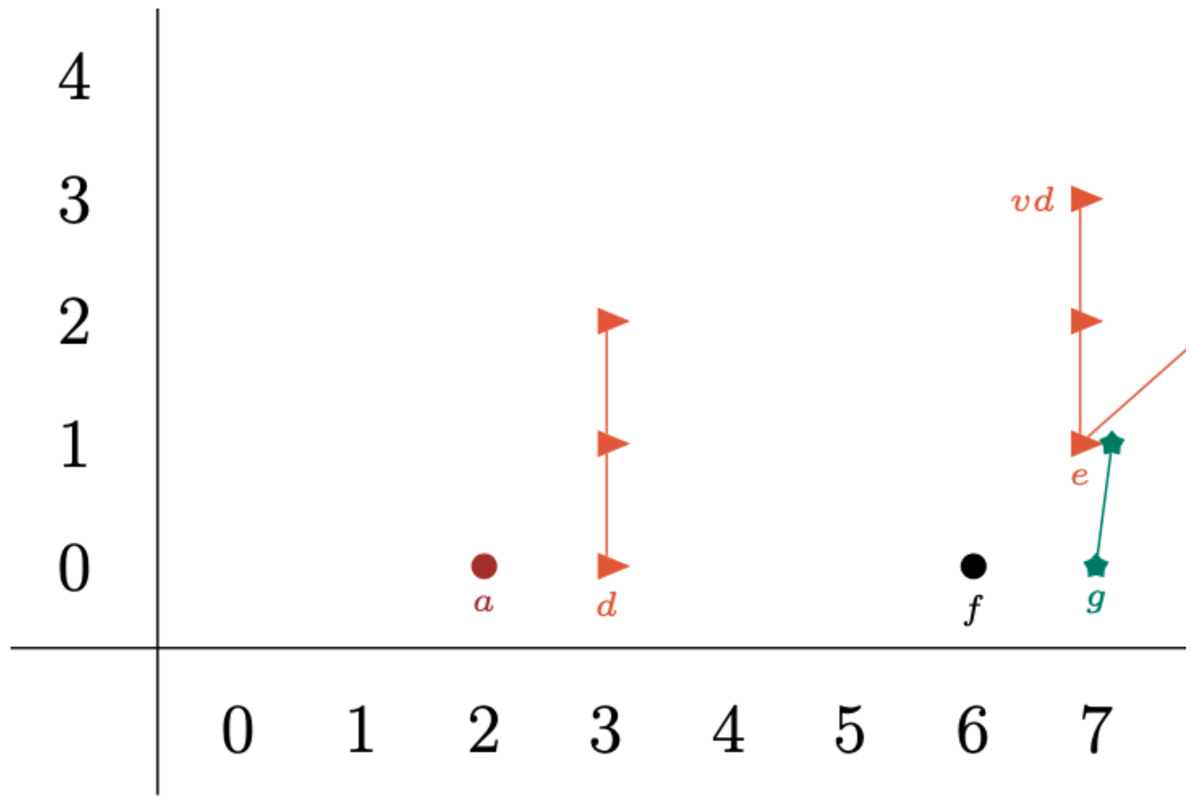
Calculation using spectral sequences

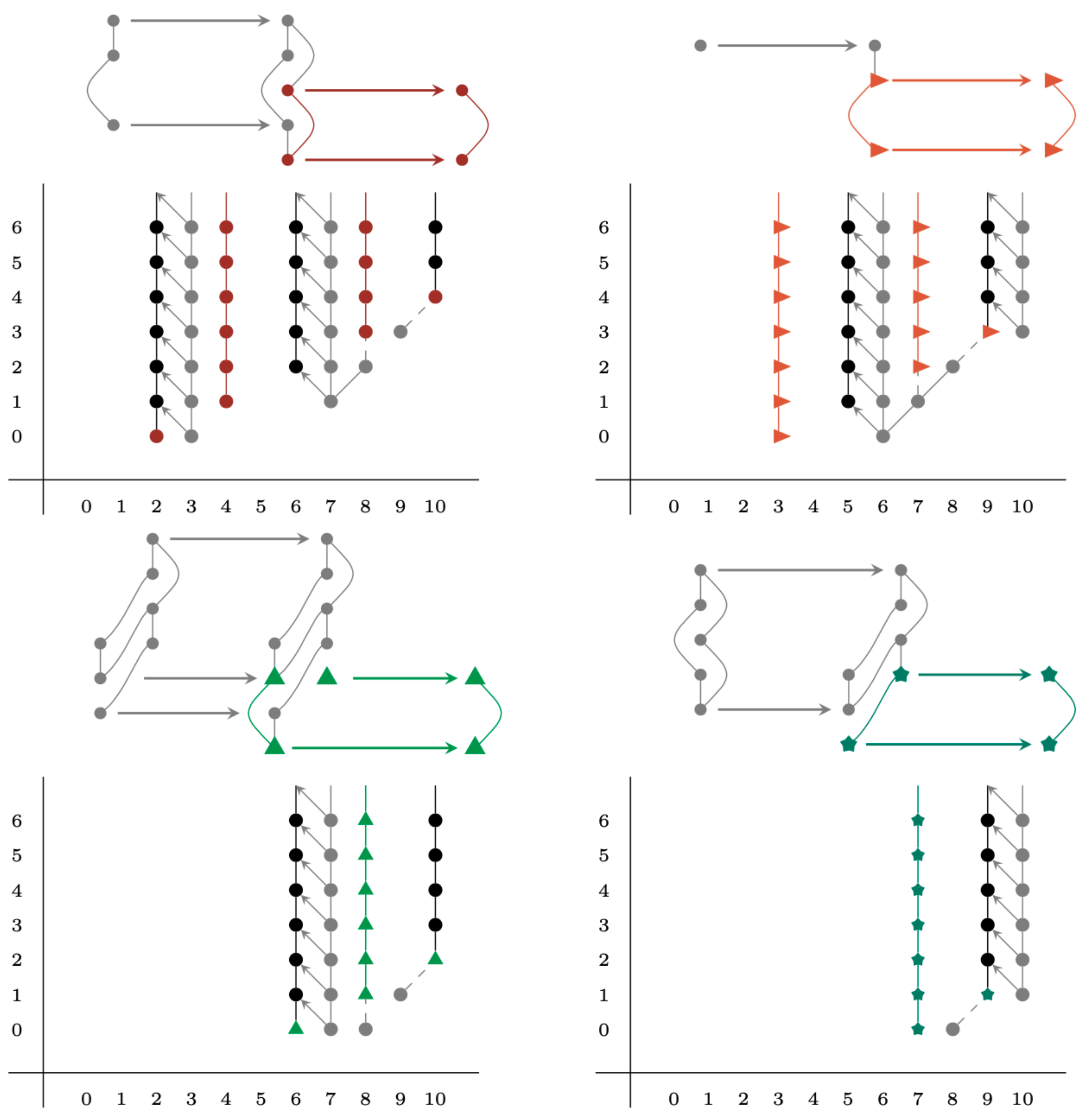
Mainly Adams (at prime 2),

(Also Atiyah Hirzebruch and homotopy equivalences to other spectra)









we have calculated the bordism groups

d	$\Omega_d^{\text{Spin}}(B\text{SL}(2,\mathbb{Z}))$ §4	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$ §5	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$ §6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

many potential global symmetries

many potential anomalies

[Debray, MD, Heckman, Montero '21 & '23]

d	$\Omega_d^{\mathrm{Spin}}(B\mathrm{SL}(2,\mathbb{Z}))$ §4	$\Omega_d^{\mathrm{Spin\text{-}Mp}(2,\mathbb{Z})}$ §5	$\Omega_d^{\mathrm{Spin\text{-}GL}^+(2,\mathbb{Z})}$ §6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

[Debray, MD, Heckman, Montero '21 & '23]

many potential anomalies

The duality anomaly

[Debray, MD, Heckman, Montero '21]

Determine anomaly theory $\mathscr{A}$ and evaluate

$$\mathcal{A}(X) = \eta_1^{\mathrm{RS}}(X) - 2\eta_1^{\mathrm{D}}(X) - \eta_{-3}^{\mathrm{D}}(X) - \tfrac{1}{8}\eta_-^{\mathrm{sig}}(X) + \mathrm{Arf}(X) - \tilde{\mathcal{Q}}(\breve{c})$$

$\mathbb{Z}_{27}$	L_3^{11}	➡	$\mathcal{A}(X) = \frac{1}{3}$
$\mathbb{Z}_3$	$\mathbb{HP}^2 \times L_3^3$	➡	$\mathcal{A}(X) = \frac{1}{3}$
$\mathbb{Z}_8$	Q_4^{11}	➡	$\mathcal{A}(X) = \frac{k}{4}$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times L_4^3$	➡	$\mathcal{A}(X) = \frac{1}{2}$

others vanish

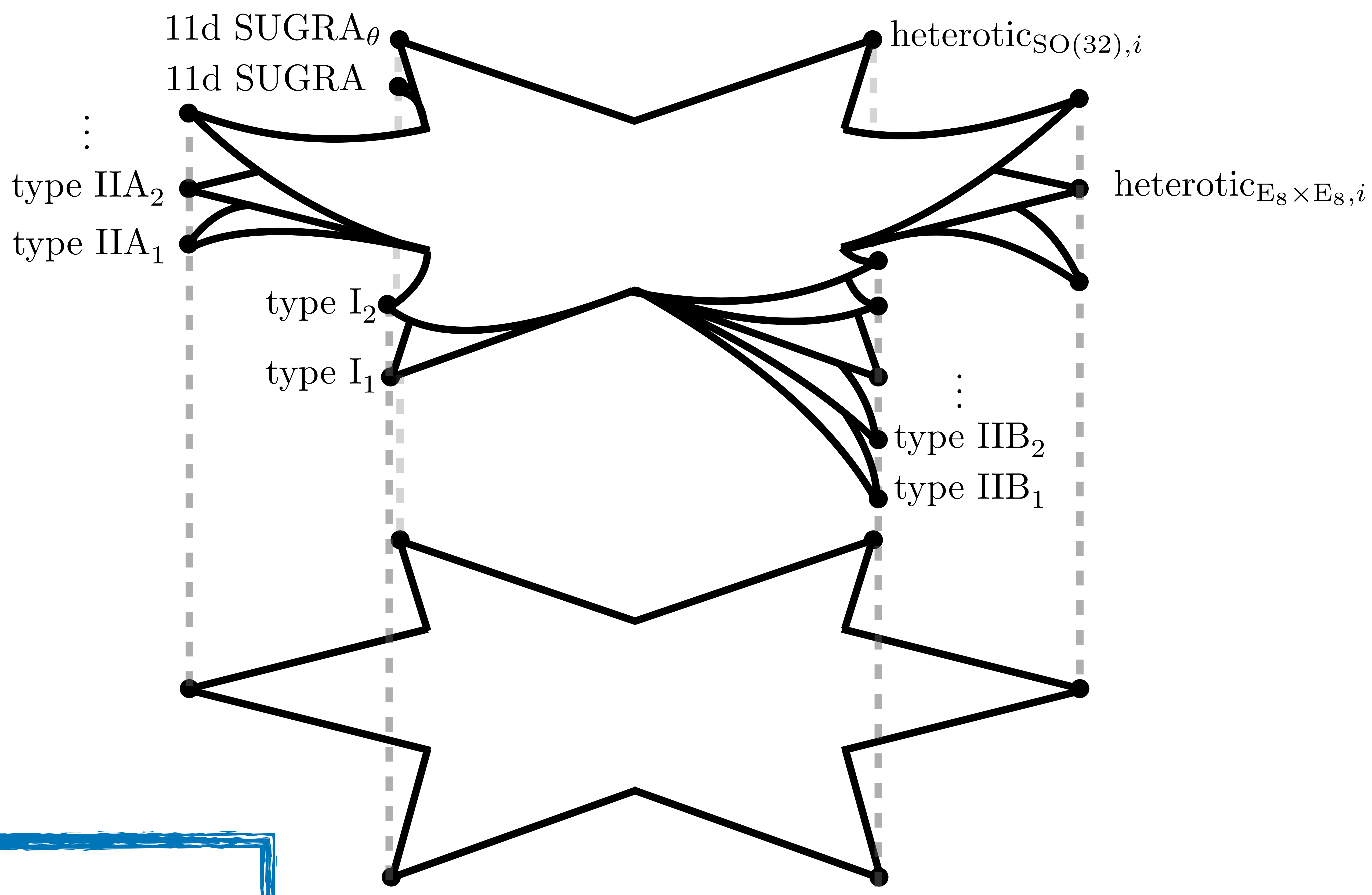
➡ Indeed the **duality has a subtle anomaly**

Cancellation

[Debray, MD, Heckman, Montero ’21]

Anomalies can be cancelled by:

- **Modification of the 4-form field** (its Bianchi identity)
- **New topological degrees of freedom**



→ **New term in the action of type IIB**

Discrete Landscape vs. Topological Swampland

d	$\Omega_d^{\text{Spin}}(B\text{SL}(2,\mathbb{Z}))$ §4	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$ §5	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$ §6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

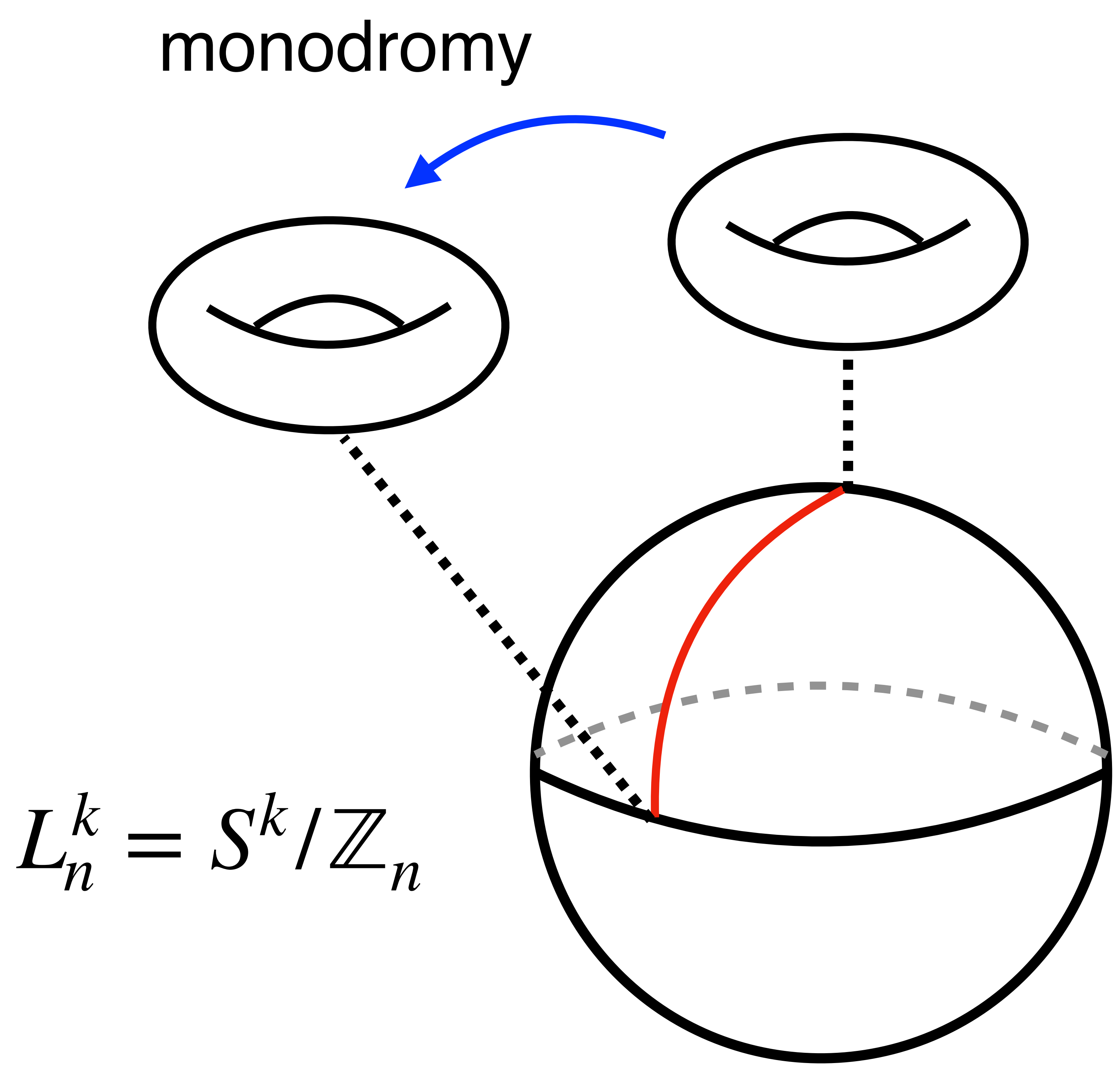
many potential global symmetries

[Debray, MD, Heckman, Montero '21 & '23]

Symmetry-breaking defects

String theory takes care of it ➔ interesting corners

Typically:



boundary of $(T^2 \times \mathbb{C}^j)/\mathbb{Z}_n$

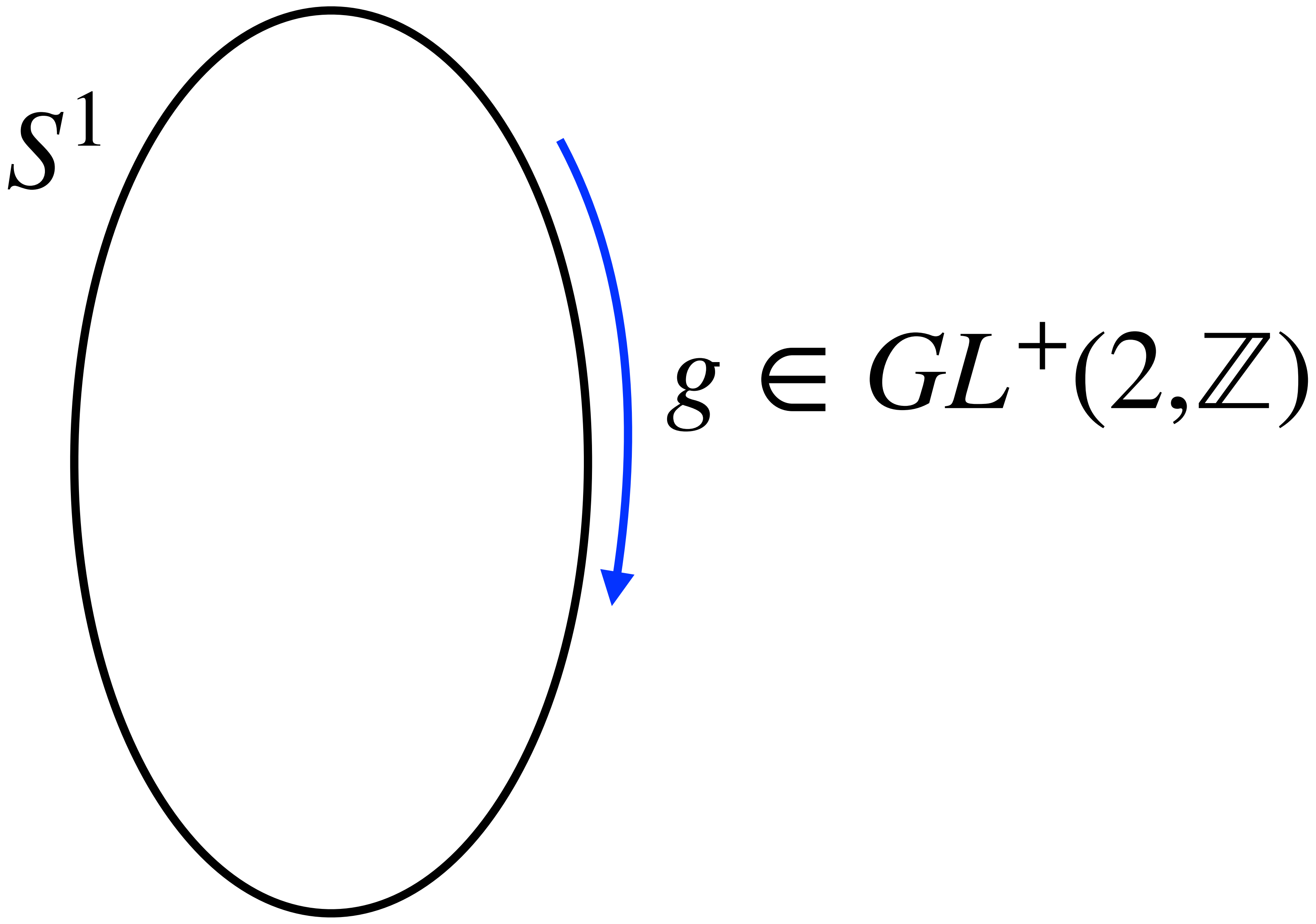
- **Non-Higgsable clusters**
  [Morrison, Taylor ’12]
- $\mathcal{N} = 3$ **S-folds**
  [Garcia-Etxebarria, Regalado ’15]
- **7-branes**
- **Topologically twisted theories**

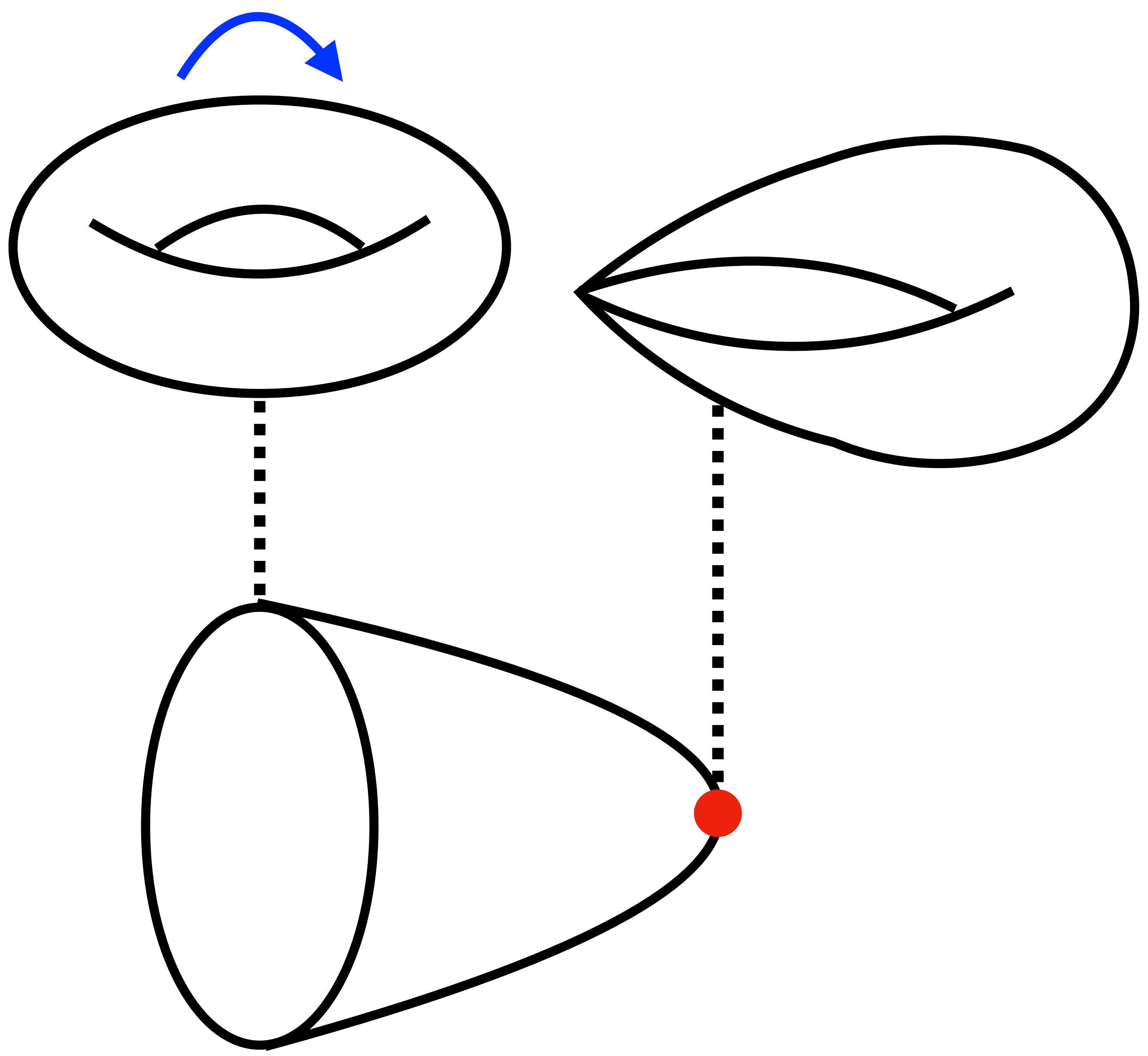
k = 1: 7-branes

$$\Omega_1^{Spin-GL^+(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

taken care of by [p,q]-7-branes

(F-theory) **see also [MD, Heckman '20]**



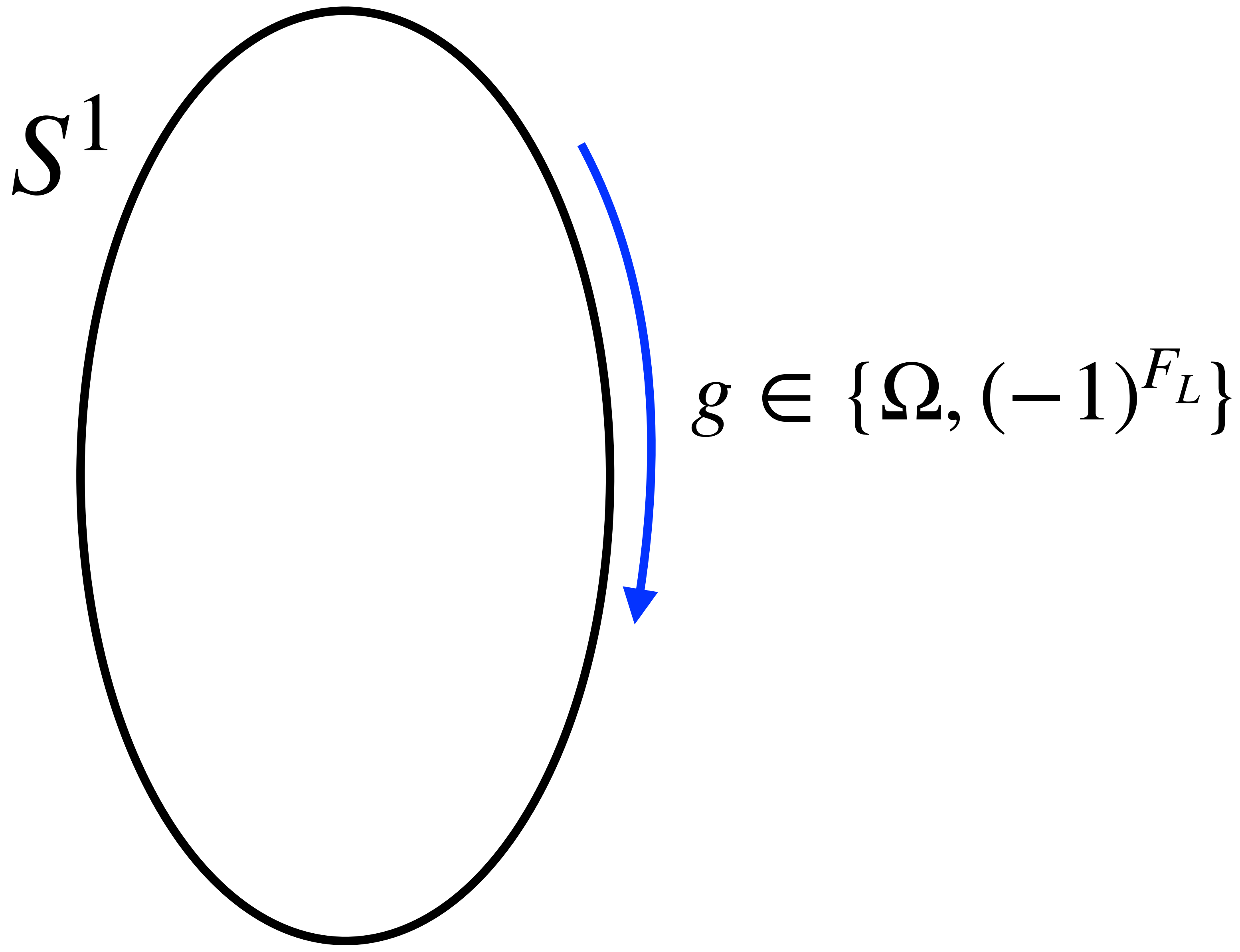


k = 1: 7-branes

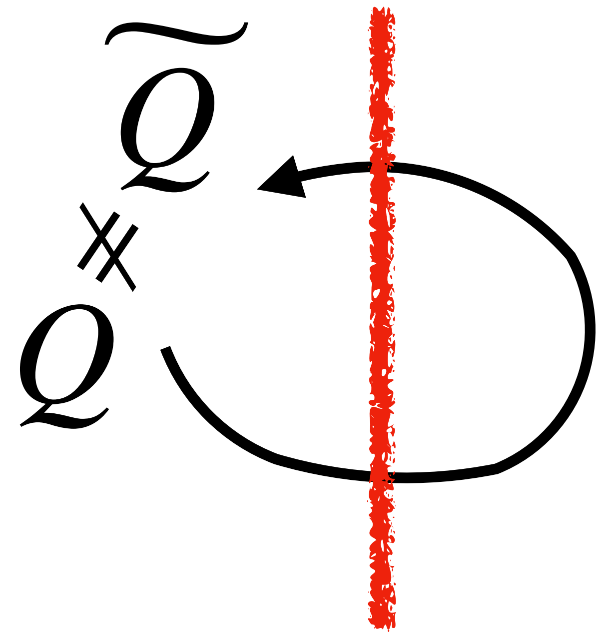
$$\Omega_1^{Spin-GL^+(2,\mathbb{Z})}(pt) = \mathbb{Z}_2 \oplus \mathbb{Z}_2$$

new 'reflection' 7-brane

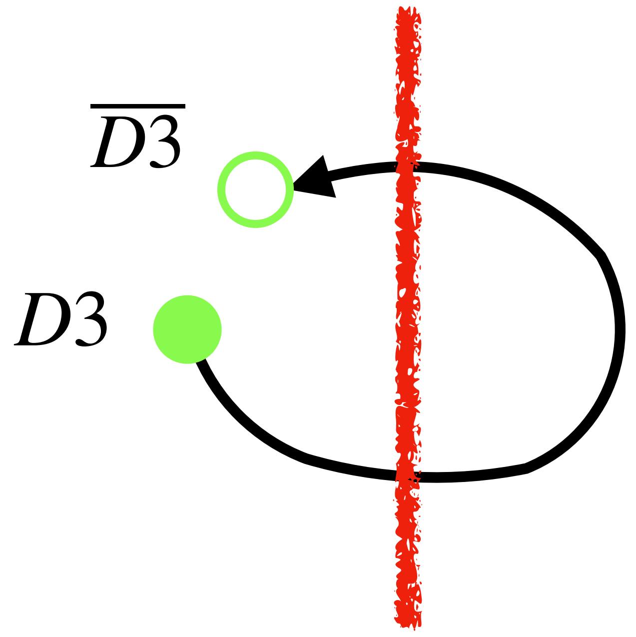
hinted at in [Distler, Freed, Moore '09]



Breaks supersymmetry

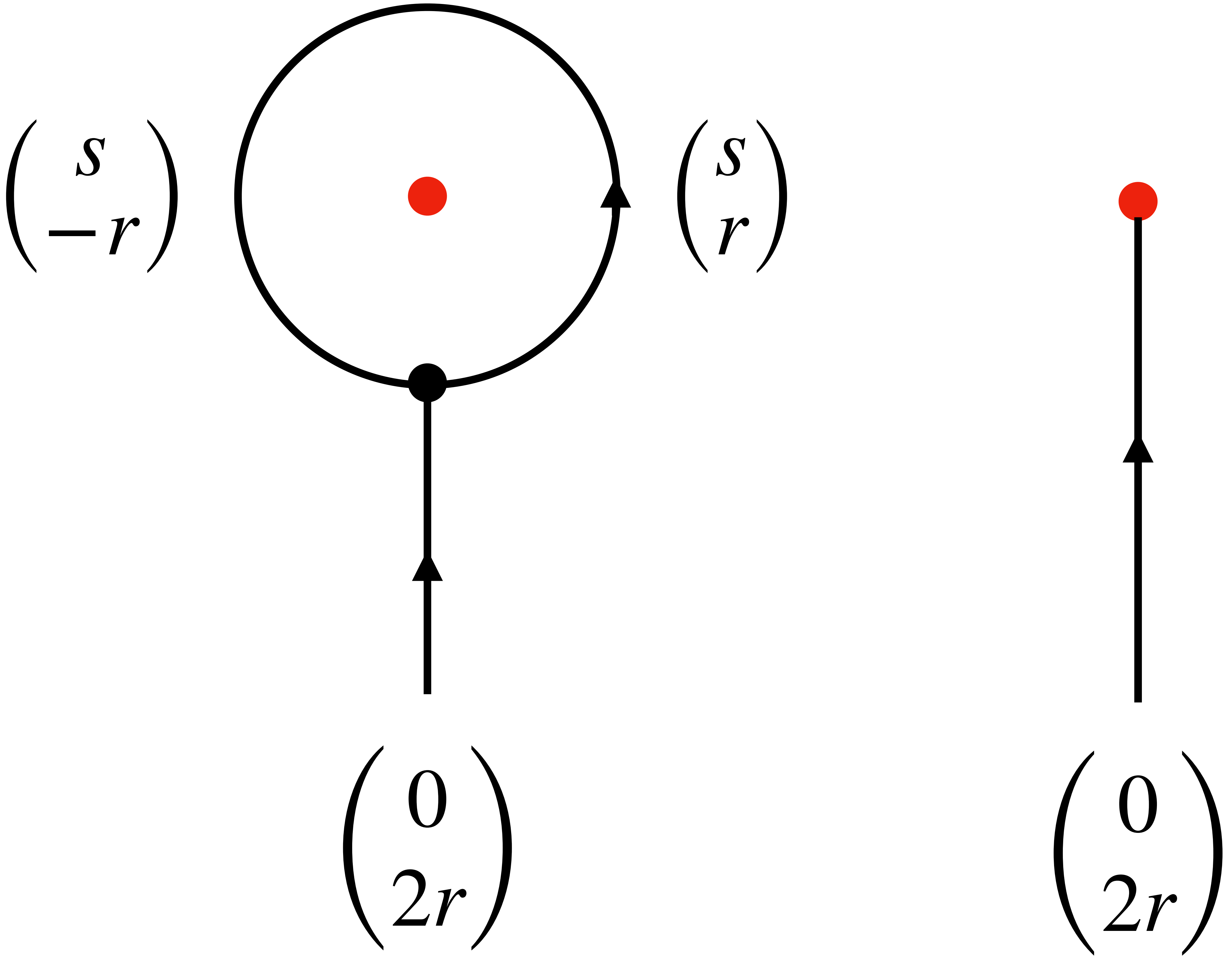


Alice string for D3 branes



Strings can end on R7-brane

[MD, Heckman, Montero, Torres '22]



F1-strings end on Ω brane

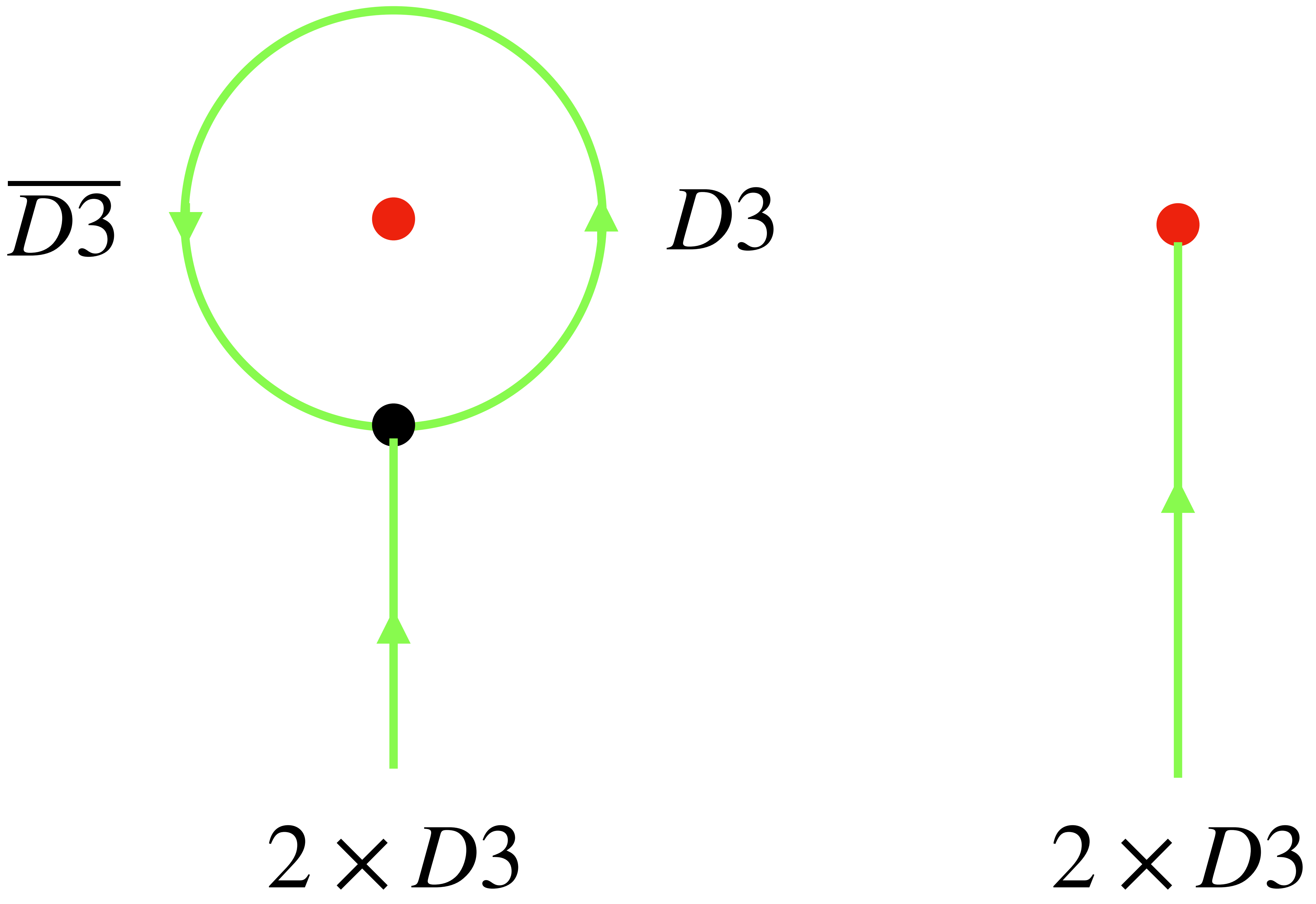
D1-strings end on $(-1)^{F_L}$ brane

(at least in pairs)

➡ something, e.g., gauge fields, should absorb charge

See also [Cvetic, MD, Lin, Zhang '21, '22] for [p,q]-7-branes

D3-branes can end on R7-brane

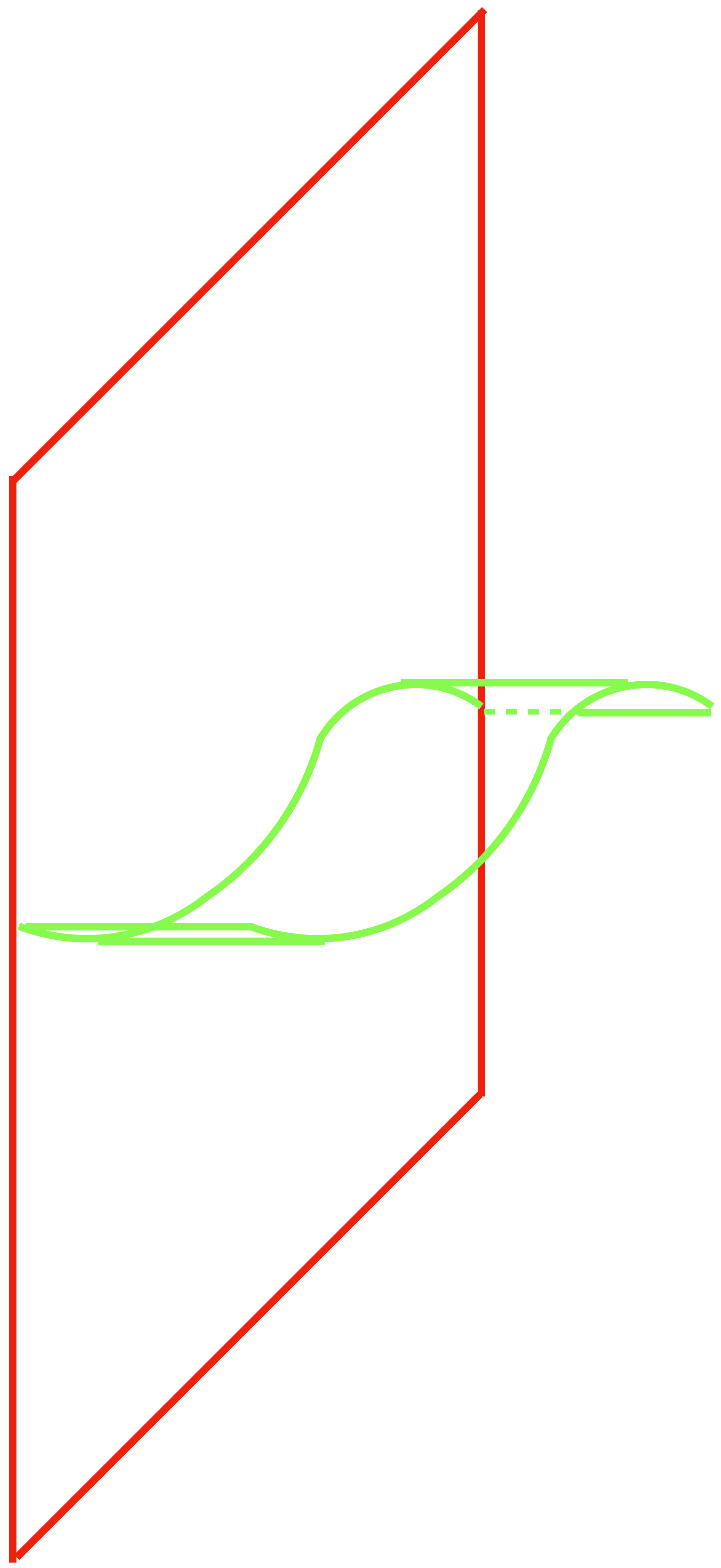


D3-branes end because of $C_4 \to -C_4$ transformation

(at least in pairs)

➡ something should absorb charge

3-form fields



D3-brane creates 3d worldvolume in R7-brane

➔ flux on transverse S^4

suggests $F_4 = dC_3$ (odd under reflections)

➔ massless 3-form on R7-brane

(potentially interesting behavior under S-duality;
interacting non-supersymmetric CFT in 8d???)

Bordisms in Quantum Gravity:

Powerful tool to find:

New consistency conditions (anomalies)

New objects (breaking global symmetries)

Surprises even for well-understood theories

a lot more to be discovered

Other works

- **Extension to U-dualities for 8d supergravity**
  [Braeger, Debray, MD, Heckman, Montero '25]

- **Discrete anomaly cancellation in 6d supergravity and its F-theory realizations**
  [MD, Oehlmann, Schimannek '22 + to appear], [MD, Tartaglia '25]
- **Anomalies for generalized symmetries and implications for string universality**
  [Cvetic, MD, Lin, Zhang '20 + '21 + '21 + '22]
- **Implications for axion physics**
  [MD, Novicic '24]

Conclusions and Outlook

Bordisms in physics:

- **Anomalies** (with some quantum gravity flavor)
- **Topological charges and their global symmetries**
- **Symmetry-protected topological order** (condensed matter theory)

What's next:

- **Extend our tools to more general symmetries**
- **Access more detailed information of symmetry-breaking defects**
- **Find a way to go beyond topological information**