Sheet 9

Question 9.1

Compute compactly supported cohomology of the open Möbius band M with coefficients in a constant sheaf. (M is defined as the quotient of $[0,1] \times (0,1)$ by the relation $(0,t) \sim (1,1-t)$.)
Can you compute it in two different ways?

Question 9.2

Define the *dimension* of a locally closed space X as

$$\dim X = \sup_{\mathcal{F} \in \mathsf{Sh}(X)} \sup_{n} H^n_c(X, \mathcal{F}) \neq 0$$

(a) Let Z be a locally closed subspace of X. Show that $\dim Z \leq \dim X$.

(b) Assuming that $\dim \mathbb{R}^n = n$ show that any real compact n-manifold has dimension n.

(c) Let $0 \to \mathcal{F} \to S^0 \to \cdots \to S^{n-1} \to T \to 0$ be an exact complex of sheaves on X with $\dim X = n$ where each S^i is Γ_c-acyclic. Show that T is also Γ_c-acyclic..

(d) Let $f : Y \to X$ be a continuos map such that $\dim X = m$ and for all $x \in X$ we have $\dim f^{-1}(x) \leq n$. Then $\dim Y \leq m + n$.

Question 9.3

Compute the sheaf cohomology with constant coefficients of the Warsaw circle from Question 5.5.

These questions will be discussed in the exercise class on 20 June 2025.

Questions with an asterisk are more challenging.