

Sheet 7

Question 7.1

Let X be a not necessarily connected space X and R a (discrete) ring. Determine $\Gamma_c(X, \underline{R})$.

Question 7.2

Consider the complex of sheaves $\Omega := \mathcal{C}^\infty \xrightarrow{\frac{d}{dx}} \mathcal{C}^\infty$ on \mathbb{R}^1 .

- (a) Compute $\Gamma(\mathbb{R}^1, \Omega)$.
- (b) Compute $\Gamma_c(\mathbb{R}^1, \Omega)$.

Question 7.3

Let $j : U \rightarrow X$ be an inclusion of locally compact Hausdorff spaces and \mathcal{F} a sheaf on U . Show that the proper pushforward $j_!\mathcal{F}$ is given by “extension by zero”, i.e. $(j_!\mathcal{F})_x = \mathcal{F}_x$ if $x \in U$ and 0 otherwise.

Question 7.4

Consider maps of topological spaces $f : X \rightarrow Y$ and $g : Z \rightarrow Y$. Denote by $f' : X \times_Y Z \rightarrow Z$ and $g' : X \times_Y Z \rightarrow X$ the natural maps from the pullback. Show that $g^{-1}f_*\mathcal{F}$ is not isomorphic to $(f')_*(g')^{-1}\mathcal{F}$ in general.

These questions will be discussed in the exercise class on 6 June.

Questions with an asterisk are more challenging.