Julian Holstein

Sheet 13

Question 13.1

(a) Let $i : X \to Y$ be an inclusion of manifolds with $\omega_{X/Y} \simeq \underline{R}_X[k]$ where k is the codimension of X. Show there is a long exact sequence of cohomology with coefficients in $\underline{\mathbb{Z}}$:

$$\cdots \to H^i(X) \to H^{i+k}(Y) \to H^{i+k}(Y \setminus X) \to H^{i+1}(X) \to \cdots$$

(b) Write down this long exact sequence for the canonical inclusion $\mathbb{C}P^m \to \mathbb{C}P^n$.

(c) When is there such a long exact sequence for the canonical inclusion $\mathbb{R}P^m \to \mathbb{R}P^n$?

Question 13.2

Given a pullback square of locally compact spaces

$$\begin{array}{ccc} \tilde{Y} & \xrightarrow{\tilde{f}} & Z \\ \downarrow{\scriptstyle \tilde{g}} & & \downarrow{\scriptstyle g} \\ Y & \xrightarrow{f} & X \end{array}$$

with f of finite cohomological dimension, show that for any complex F in $D(Z)$ there is a canonical quasi-isomorphism

$$f^! \circ Rg_* F \simeq R\tilde{g}_* \tilde{f}^! F.$$

Question 13.3

Given $f : X \to Y$ and R with the standing assumptions and a a complex G in $D(\underline{R}_Y)$ construct a map $f^! \underline{R}_Y \otimes f^{-1} G \to f^! G$.

Question 13.4

Let k be a field. Show that the category of locally constant sheaves of k-vector spaces on a space X is equivalent to the category of k-representations of $\pi_1(X, x)$.

These questions will be discussed in the exercise class on 18 July 2025.

Questions with an asterisk are more challenging.