Julian Holstein

Sheet 11

Question 11.1

(a) Describe explicitly all real line bundles on S^1.

(b) Describe explicitly all real line bundles on $\mathbb{R}P^n$.

(c) Describe explicitly all complex line bundles on S^2.

Question 11.2

(a) Describe a sheaf cohomology group classifying all locally constant sheaves with fiber $\mathbb{C}$ on a manifold M.

(b) Compute this group if $M = \mathbb{R}P^2$.

(c) Compute this group if M is an oriented surface with genus g.

Question 11.3

(a) Show that fine sheaves are c-soft.

(b) Define compactly supported de Rham cohomology and show that it computes compactly supported cohomology with coefficients in $\underline{\mathbb{R}}$.

(c) Exhibit a generator for $H^n_{c,dR}(\mathbb{R}^n, \mathbb{R})$, starting with $n = 1$.

Question 11.4

Consider the sheaf $S = \oplus_{n\in\mathbb{Z}}\mathbb{R}_n$ on $\mathbb{R}$ where $\mathbb{R}_n$ is a skyscraper sheaf with fiber $\mathbb{R}$ at the point n.

Compute $H^*(\mathbb{R}, S)$ and $H^*_c(\mathbb{R}, S)$.

Compute $H^*(\mathbb{R}^2, S \boxtimes S)$ and $H^*_c(\mathbb{R}^2, S \boxtimes S)$ and compare the result with $H^*(\mathbb{R}, S) \otimes H^*(\mathbb{R}, S)$ and $H^*_c(\mathbb{R}, S) \otimes H^*_c(\mathbb{R}, S)$, respectively.

These questions will be discussed in the exercise class on 4 July 2025.

Questions with an asterisk are more challenging.